Algebraic Structure Of Union Of Fuzzy Sub-Trigroups And Fuzzy Sub-Ngroups

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ABSTRACT: In this paper, the definitions of union of Fuzzy subsets, Fuzzy subgroup, properties and union of Fuzzy subgroups. Some characteristic description and a kind of representation of the fuzzy sub-trigroup with respect to operation addition are Introduced. Fuzzy sub-trigroup, union of fuzzy trigroups is proved with examples.

Index Terms: Fuzzy group, Fuzzy sub-bigroup , Fuzzy subgroup, Trigroup, Fuzzy sub-trigroup .

INTRODUCTION:
The concepts of fuzzy sets was introduced by Zadeh. Since the paper fuzzy set theory has been considerably developed by Zadeh himself and some researchers. The original concept of fuzzy sets was introduced as an extension of crisps (usual) sets, by enlarging the truth value set of “grade of members” from the two value set {0,1} to unit interval [0,1] of real numbers. The study of fuzzy group was started by Rosenfeld. It was extended by Roventa who have introduced the fuzzy groups operating on fuzzy sets. W.B.Vasanthakandasamy introduced fuzzy sub-bigroup with respect to “ + and _” and illustrate it with example. W.B.VasanthaKandasamy was the first one to introduce the notion of bigroups in the year 1994. Several mathematicians have followed them in investigating the fuzzy group theory. We now recall the previous and preliminary definitions, and results that are required in our discussion. Rosenfield introduced the notion of fuzzy group and showed that many group theory results can be extended in an elementary manner to develop the theory of fuzzy group. The underlying logic of the theory of fuzzy group is to provide a strict fuzzy algebraic structure where level subset of a fuzzy group of a group G is a subgroup of the group. In this section, We define Fuzzy sub-bigroup of a bigroup. To define the notion of fuzzy sub-bigroup of a bigroup. We define a new notion called the fuzzy union of any two fuzzy subsets of two distinct sets.

2. Preliminaries:

Concept of a Fuzzy set: 1
The concept of a fuzzy set is an extension of the concept of a crisp set. Just as a crisp set on a universal set U is defined by its characteristic function from U to {0,1}, a fuzzy set on a domain U is defined by its membership function from U to [0,1]. Let U be a non-empty set, to be called the Universal set or Universe of discourse or simply a domain. Then, by a fuzzy set on U is meant a function. A: U → [0,1]. A is called the membership function; A(x) is called the membership grade of x in A. We also write
A = {(x, A(x)): x ∈ U}.

Examples:
Consider U={a,b,c,d} and A:U→I defined by A(a)=0, A(b)=0.7, A(c)=0.4, and A(d)=1. Then A is a fuzzy set can also be written as follows:
A= {(a, 0), (b,0.7),(c,0.4),(d,1)}.

Relation between Fuzzy sets: 2
Let U be a domain and A, B be fuzzy sets on U. Inclusion (or) Containment: A is said to be included (or) contained in B if and only if A(x) ≤ B(x) for all x in U. In symbols, we write, A⊆B. Also we say that A is a subset of B.

Definition of Union of Fuzzy sets: 3
The union of two fuzzy subsets μ1 and μ2 is defined by
μA (x) = max {μ1(x), μ2(x)} for every x in U.

Definition of Fuzzy Subgroup: 4
Let G be a group. A fuzzy subset μ of a group G is called a fuzzy subgroup of the group G if
i) μ(xy) ≥ min {μ(x), μ(y)} for every x,y ∈ G and
ii) μ(x⁻¹) = μ(x) for every x ∈ G.

Definition of Fuzzy Union of the fuzzy sets μ1 and μ2: 5
Let μ1 be a fuzzy subset of a set x and μ2 be a fuzzy subset of a set y, then the fuzzy union of the fuzzy sets μ1 and μ2 is defined as a function.

\[ μ_1 \cup μ_2 : x_1 \cup x_2 \to [0,1] \text{ given by} \]
\[ (μ_1 \cup μ_2)(x) = \begin{cases} \max (μ_1(x), μ_2(x)) & \text{if } x \in x_1 \cap x_2, \\ μ_1(x) & \text{if } x \in x_1 \setminus x_2, \\ μ_2(x) & \text{if } x \in x_2 \setminus x_1. \end{cases} \]

We illustrate this definition by the following example:
Let X₁ = {1,2,3,4,5} and X₂ = {2,4,6,8,10} be two sets.
Define \[ μ_1 : X_1 \to [0,1] \text{ by} \]
\[ μ_1(x) = \begin{cases} 1 & \text{if } x = 1, 2, \text{ or } 3, \\ 0.6 & \text{if } x = 3. \end{cases} \]
and define $\mu_2 : X_2 \rightarrow [0,1]$ by

$$
\mu_2(x) = \begin{cases} 
1 & \text{if } x = 2, 4, \\
0.6 & \text{if } x = 6, \\
0.2 & \text{if } x = 8, 10.
\end{cases}
$$

It is easy to calculate $\mu_1 \cup \mu_2$ and it is given as follows:

$$(\mu_1 \cup \mu_2)(x) = \begin{cases} 
1 & \text{if } x = 1, 2, 4, \\
0.6 & \text{if } x = 3, 6, \\
0.2 & \text{if } x = 5, 8, 10.
\end{cases}
$$

Definition of Fuzzy sub-bigroup : 6

Let $G = (G_1 \cup G_2, +, \cdot)$ be a bigroup. Then $\mu : G \rightarrow [0,1]$ is said to be a Fuzzy sub-bigroup of the bigroup $G$ if there exists two fuzzy subsets $\mu_1$ of $G_1$ and $\mu_2$ of $G_2$ such that

i) $(\mu_1, +)$ is a fuzzy subgroup of $(G_1, +)$

ii) $(\mu_2, \cdot)$ is a fuzzy subgroup of $(G_2, \cdot)$

iii) $\mu = \mu_1 \cup \mu_2$.

We illustrate this by the example:

Consider the bigroup $G = \{i, 0, 1, 2, 3, 4, 5\}$ under the binary operation $+$ and $\cdot$ where $G_1 = \{0, 1, 2, 3, 4, 5\}$ and $G_2 = \{i, 0, 1\}$. Define $\mu : G \rightarrow [0,1]$ by

$$
\mu(x) = \begin{cases} 
\frac{1}{3} & \text{if } x = i, -i \\
\frac{1}{2} & \text{if } x \in \{0, \pm 2, \pm 4, \ldots\} \\
\frac{1}{2} & \text{if } x \in \{\pm 1, \pm 3, \ldots\}
\end{cases}
$$

It is easy to verify that $\mu$ is a fuzzy sub-bigroup of the bigroup $G$, for we can find $\mu_1 : G_1 \rightarrow [0,1]$ by

$$
\mu_1(x) = \begin{cases} 
\frac{1}{2} & \text{if } x \in \{0, \pm 2, \pm 4, \ldots\} \\
\frac{1}{2} & \text{if } x \in \{\pm 1, \pm 3, \ldots\}
\end{cases}
$$

and $\mu_2 : G_2 \rightarrow [0,1]$ is given by

$$
\mu_2(x) = \begin{cases} 
\frac{1}{2} & \text{if } x = 1, -1 \\
\frac{1}{3} & \text{if } x = i, -i
\end{cases}
$$

i.e., there exists two fuzzy subgroup $\mu_1$ of $G_1$ and $\mu_2$ of $G_2$ such that $\mu = \mu_1 \cup \mu_2$.

Definition of the fuzzy union of the fuzzy sets $\mu_1$ and $\mu_2$, $\mu_3$ : 7 (addition)

Let $\mu_1$ be a fuzzy subset of a set $X_1$ and $\mu_2$ be a fuzzy subset of a set $X_2$, $\mu_3$ be a fuzzy subset of a set $X_3$, then the fuzzy union of the fuzzy sets $\mu_1$, $\mu_2$, $\mu_3$ is defined as a function.

$$
\mu_1 \cup \mu_2 \cup \mu_3 : X_1 \cup X_2 \cup X_3 \rightarrow [0,1] \text{ given by }
$$

$$
(\mu_1 \cup \mu_2 \cup \mu_3)(x) = \begin{cases} 
\max(\mu_1(x), \mu_2(x), \mu_3(x)) & \text{if } x \in X_1 \cap X_2 \cap X_3 \\
\mu_1(x) & \text{if } x \in X_1 \cap x \notin X_2, X_3 \\
\mu_2(x) & \text{if } x \in X_2 \cap x \notin X_1, X_3 \\
\mu_3(x) & \text{if } x \in X_3 \cap x \notin X_1, X_2.
\end{cases}
$$

Definition of fuzzy sub-trigroup of the trigroup $G : 8$

Let $G = (G_1 \cup G_2 \cup G_3, +)$ be a trigroup. Then $\mu : G \rightarrow [0,1]$ is said to be a fuzzy sub-trigroup of the trigroup $G$ if there exists three fuzzy subsets $\mu_1$ of $G_1$, $\mu_2$ of $G_2$, $\mu_3$ of $G_3$ such that

i) $(\mu_1, +)$ is a fuzzy subgroup of $(G_1, +)$

ii) $(\mu_2, +)$ is a fuzzy subgroup of $(G_2, +)$

iii) $(\mu_3, +)$ is a fuzzy subgroup of $(G_3, +)$

iv) $\mu = (\mu_1 \cup \mu_2 \cup \mu_3)$.

Note: A fuzzy subset $\mu$ of a group $G$ is said to be a union of three fuzzy sub-groups of the group $G$ if there exists three fuzzy subgroups $\mu_1$ and $\mu_2$ and $\mu_3$ of $\mu$ such that $\mu = \mu_1 \cup \mu_2 \cup \mu_3$. Here by the term fuzzy subgroup $\lambda$ of $\mu$ we mean that $\lambda$ is a fuzzy subgroup of the group $G$ and $\lambda \subseteq \mu$ (where $\mu$ is also a fuzzy subgroup of $G$).

Example:

Let $X_1 = \{1, 2, 3, 4, 5\}$ and $X_2 = \{2, 4, 6, 8, 10\}$, $X_3 = \{1, 3, 5, 7, 9\}$ be three sets

Define $\mu_1 : X_1 \rightarrow [0,1]$ by

$$
\mu_1(x) = \begin{cases} 
1 & \text{if } x = 1, 2, 4, 5 \\
0.6 & \text{if } x = 3, 7, 9 \\
0.2 & \text{if } x = 6, 8, 10.
\end{cases}
$$

Define $\mu_2 : X_2 \rightarrow [0,1]$ by

$$
\mu_2(x) = \begin{cases} 
1 & \text{if } x = 1, 2, 4, 5 \\
0.6 & \text{if } x = 3, 7, 9 \\
0.2 & \text{if } x = 6, 8, 10.
\end{cases}
$$

And Define $\mu_3 : X_3 \rightarrow [0,1]$ by

$$
\mu_3(x) = \begin{cases} 
1 & \text{if } x = 1, 2, 4 \\
0.6 & \text{if } x = 3, 7, 9 \\
0.2 & \text{if } x = 6, 8, 10.
\end{cases}
$$

Hence

$$(\mu_1 \cup \mu_2 \cup \mu_3)(x) = \begin{cases} 
1 & \text{if } x = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.
\end{cases}
$$
Definition of the fuzzy union of the fuzzy sets $\mu_1$ and $\mu_2$, $\mu_3$, $\mu_4$: 9 (addition)

Let $\mu_1$ be a fuzzy subset of a set $X_1$ and $\mu_2$ be a fuzzy subset of a set $X_2$, $\mu_3$ be a fuzzy subset of a set $X_3$, $\mu_4$ be a fuzzy subset of a set $X_4$. Then the fuzzy union of the fuzzy sets $\mu_1$ and $\mu_2$, $\mu_3$, $\mu_4$ is defined as a function.

$$\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 : X_1 \cup X_2 \cup X_3 \cup X_4 \to [0,1]$$

is given by

$$\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 (x) = \begin{cases} 
\max (\mu_1 (x), \mu_2 (x)) & \text{if } x \in X_1 \\
\mu_3 (x) & \text{if } x \in X_2 \setminus X_1 \\
\mu_4 (x) & \text{if } x \in X_3 \setminus X_1 \\
\mu_5 (x) & \text{if } x \in X_4 \setminus X_1 \\
\min \{ \mu_1 (x), \mu_2 (x), \mu_3 (x), \mu_4 (x) \} & \text{if } x \in X_1 \cap X_2 \cap X_3 \cap X_4.
\end{cases}$$

Fuzzy sub-quadratic group of a quadratic group $G$: 10 (addition)

Let $G = (G_1 \cup G_2 \cup G_3 \cup G_4, +)$ be a quadratic group. Then $\mu : G \to [0,1]$ is said to be a fuzzy sub-quadratic group of the quadratic group $G$ if there exists three fuzzy subsets $\mu_1$ of $G_1$ and $\mu_2$ of $G_2$, $\mu_3$ of $G_3$, $\mu_4$ of $G_4$, such that

(i) $(\mu_1, +)$ is a fuzzy subgroup of $(G_1, +)$
(ii) $(\mu_2, +)$ is a fuzzy subgroup of $(G_2, +)$
(iii) $(\mu_3, +)$ is a fuzzy subgroup of $(G_3, +)$
(iv) $(\mu_4, +)$ is a fuzzy subgroup of $(G_4, +)$
(v) $\mu = (\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4)$.

Similarly, we have in the following for $n$,

Definition of the fuzzy union of the fuzzy sets $\mu_1$ and $\mu_2$, $\mu_3$, $\mu_4$,..., $\mu_n$: 11 (addition)

Let $\mu_1$ be a fuzzy subset of a set $X_1$ and $\mu_2$ be a fuzzy subset of a set $X_2$. $\mu_3$ be a fuzzy subset of a set $X_3 \ldots$, $\mu_n$ be a fuzzy subset of a set $X_n$. Then the fuzzy union of the fuzzy sets $\mu_1$ and $\mu_2$, $\mu_3$, $\mu_4$ is defined as a function.

$$\mu_1 \cup \mu_2 \cup \ldots \cup \mu_n : X_1 \cup X_2 \cup \ldots \cup X_n \to [0,1]$$

is given by

$$\mu_1 \cup \mu_2 \cup \ldots \cup \mu_n (x) = \begin{cases}
\max (\mu_1 (x), \mu_2 (x), \mu_3 (x), \ldots, \mu_n (x)) & \text{if } x \in X_1 \cap X_2 \ldots \cap X_n, \\
\mu_1 (x) & \text{if } x \in X_2 \setminus X_1 \ldots \setminus X_{n-1}, \\
\mu_2 (x) & \text{if } x \in X_3 \setminus X_1 \ldots \setminus X_{n-2}, \\
\mu_3 (x) & \text{if } x \in X_4 \setminus X_1 \ldots \setminus X_{n-3}, \\
\mu_4 (x) & \text{if } x \in X_5 \setminus X_1 \ldots \setminus X_{n-4}, \\
\ldots & \ldots \\
\mu_n (x) & \text{if } x \in X_n \setminus X_1 \ldots \setminus X_{n-2}.
\end{cases}$$

Fuzzy sub-$n$ groups of a $n$ group $G$: 12

Let $G = (G_1 \cup G_2 \cup \ldots \cup G_n, +)$ be a $n$-group. Then $\mu : G \to [0,1]$ is said to be a fuzzy sub-$n$ groups of the $n$ group $G$. If there exists $n$ fuzzy subsets $\mu_1$ of $G_1$ and $\mu_2$ of $G_2$, $\mu_3$ of $G_3$, $\ldots$, $\mu_n$ of $G_n$, such that

(i) $(\mu_1, +)$ is a fuzzy subgroup of $(G_1, +)$
(ii) $(\mu_2, +)$ is a fuzzy subgroup of $(G_2, +)$
(iii) $(\mu_3, +)$ is a fuzzy subgroup of $(G_3, +)$
(iv) $\ldots$
(n) $(\mu_n, +)$ is a fuzzy subgroup of $(G_n, +)$.

$$\mu = (\mu_1 \cup \mu_2 \cup \mu_3 \ldots \cup \mu_n).$$

3. Theorems:

Theorem: 1

Every fuzzy sub-bigroup of a group $G$ is a fuzzy subgroup of the group $G$ but not conversely.

Proof: It follows from the definition of the fuzzy sub-bigroup of a group $G$ that every sub-bigroup of a group $G$ is a fuzzy subgroup of the group $G$.

Main Theorem: 1

The union of two fuzzy subgroups of a group $G$ is a fuzzy subgroup if and only if one is contained in the other.

Proof:

Necessary part: Let $\mu_1$ and $\mu_2$ be two fuzzy subgroups of $G$ such that one is contained in the other. Hence either $\mu_1 \subseteq \mu_2$ (or) $\mu_2 \subseteq \mu_1$.

To prove: $\mu_2$ is a fuzzy subgroup of $G$. Let the union of two fuzzy subsets $\mu_1$, $\mu_2$ be defined by

$$\mu_2 (x) = \max \{ \mu_2 (x), \mu_2 (x) \}$$

So $\mu_1 \cup \mu_2 (xy) \geq \max \{ \mu_1 (xy), \mu_2 (xy) \} \ldots \ldots \ldots (1)$

Since $\mu_1$ and $\mu_2$ are the two fuzzy subgroup of $G$.

$$\mu_2 (xy) = \mu_2 (xy) \ldots \ldots 2.$$ 

Since $\mu_1 (xy)$ and $\mu_2 (xy)$ are fuzzy subgroup of $G$.

From 1 and 2.

$$\mu_1 \cup \mu_2 (xy)$$

is also a fuzzy subgroup of $G$.

Hence $\mu_1 \cup \mu_2$ is a fuzzy subgroup of $G$.

SUFFICIENT PART:

Suppose $\mu_1 \cup \mu_2$ is a fuzzy subgroup of $G$.

To Claim:

$$\mu_1 \subseteq \mu_2 \ (or) \ \mu_2 \subseteq \mu_1.$$ 

Since $\mu_1$, and $\mu_2$ are fuzzy subgroup of $G$.

$$(xy) \geq \min \{ \mu_1 (x), \mu_1 (y) \} \ldots \ldots \ldots (by \ condition \ (i) \ of \ definition \ of \ fuzzy \ subgroup \ of \ G)).$$

There are two cases,

i) $\mu_1 (xy) \geq \mu_1 (x)$

ii) $\mu_1 (xy) \geq \mu_1 (y)$.
Case i)

\[ \mu_1(xy) \geq \mu_1(x) \]  

By fuzzy union of fuzzy sets \( \mu_1 \) and \( \mu_2 \), we have

\[ \cup \mu_2(x) = \mu_1(x) \]

Sub 5 in 4, we get

\[ (xy) \geq \mu_1 \cup \mu_2(x) = \mu_2(x) \]

From 4 and 6, we get

\[ \subseteq \]

Similarly case ii):

\[ \mu_1(xy) \geq \mu_1(y) \]

By definition of fuzzy union of fuzzy sets of G, we have

\[ \cup \mu_2(y) = \mu_1(y) \]

Sub 9 in 8, we get

\[ (xy) \geq \mu_1 \cup \mu_2(y) = \mu_2(y) \]

From 8 and 10 we have,

\[ (y) \leq \mu_1(y) \]

\[ \subseteq \]

From 7 and 11,

\[ \mu_1 \subseteq \mu_2 \]

Hence \( \mu_1 \) is contained in \( \mu_2 \) and \( \mu_2 \) is contained in \( \mu_1 \).

Therefore The union of two fuzzy subgroups of a group G is a fuzzy subgroup if and only if one contained in the other.

Main Theorem: 2

The union of two fuzzy sub-bigroups of a bigroup G is a fuzzy sub-bigroup if and only if one is contained in the other.

Proof:

Necessary part: Let \( \mu_1 \) and \( \mu_2 \), \( \mu_3 \) be three fuzzy sub-bigroups of G such that one is contained in the other.

Hence either \( \mu_1 \subseteq \mu_2 \) (or) \( \mu_2 \subseteq \mu_1 \), (or) \( \mu_3 \subseteq \mu_2 \) (or) \( \mu_3 \subseteq \mu_1 \).

To prove: \( \cup \mu_2 \cup \mu_3 \) is a fuzzy sub-bigroup of G.

By Definition of the fuzzy union of the fuzzy sets \( \mu_1 \) and \( \mu_2 \), \( \mu_3 \).

\[ \mu(x) = \max(\mu_1(x), \mu_2(x), \mu_3(x)) \]

which implies either

\[ \mu(x) = \max(\mu_1 \cup \mu_2 \cup \mu_3)(x) = \mu_1(x) \]

(1)

(2)

From 1 , 2, 3 since \( \mu_1 \) and \( \mu_2 \), \( \mu_3 \) be three fuzzy sub-bigroups.

Sufficient part:

Let \( \mu_1 \) and \( \mu_2 \) be a fuzzy sub-bigroup of G.

To prove: \( \cup \mu_1 \cup \mu_2 \) is a fuzzy sub-bigroup of G.

By the Definition of fuzzy sub-bigroup

Let \( G = (G_1 \cup G_2 \cup G_3, +) \) : \( \mu = \mu_1 \cup \mu_2 \cup \mu_3 \) and

( i ) \( \mu_1, + \) is a fuzzy subgroup of \( (G_1, +) \).

( ii ) \( \mu_2, + \) is a fuzzy subgroup of \( (G_2, +) \).

( iii ) \( \mu_3, + \) is a fuzzy subgroup of \( (G_3, +) \).
By the Note:
A fuzzy subset $\mu$ of a group $G$ is said to be a union of three fuzzy sub-groups of the group $G$ if there exists three fuzzy subgroups $\mu_1$ and $\mu_2$, $\mu_3$ of $\mu$ ($\mu_1=\mu$, $\mu_2=\mu$ and $\mu_3=\mu$) such that

$$\mu = (\mu_1 \cup \mu_2 \cup \mu_3).$$

which implies

$$\mu_1 \subseteq \mu = (\mu_1 \cup \mu_2 \cup \mu_3).$$  \hspace{1cm} \text{4} \text{ (where $\mu$ is also a fuzzy subgroup of $G$).}$$

similarly,

$$\mu_2 \subseteq \mu = (\mu_1 \cup \mu_2 \cup \mu_3).$$ \hspace{1cm} \text{5}$$

$$\mu_3 \subseteq \mu = (\mu_1 \cup \mu_2 \cup \mu_3).$$ \hspace{1cm} \text{6}.$$

By the fuzzy union of the fuzzy sets $\mu_1$ and $\mu_2$, $\mu_3$:

$$\mu_1 \cup (\mu_2 \cup \mu_3). (x) =
\begin{cases} 
\mu_1 (x) & \text{if } x \in X_1 \text{ & } x \notin X_2, X_3 \\
\mu_2 (x) & \text{if } x \in X_2 \text{ & } x \notin X_1, X_3 \\
\mu_3 (x) & \text{if } x \in X_3 \text{ & } x \notin X_1, X_2
\end{cases}
$$

Suppose that

$$\mu_1 \cup (\mu_2 \cup \mu_3). (x) = \mu_1 (x) \hspace{1cm} \text{7}.$$  

$$\mu_2 \cup (\mu_2 \cup \mu_3). (x) = \mu_2 (x) \hspace{1cm} \text{8}.$$ 

$$\mu_3 \cup (\mu_2 \cup \mu_3). (x) = \mu_3 (x) \hspace{1cm} \text{9}.$$ 

Substitute 5, 6 in 4 We get,

$$\mu_1 \subseteq \mu_2 \text{ (or) } \mu_1 \subseteq \mu_3 \hspace{1cm} \text{I}.$$  

Similarly Substitute 4 and 6 in 5, We get,

$$\mu_2 \subseteq \mu_1 \text{ (or) } \mu_2 \subseteq \mu_3 \hspace{1cm} \text{II}.$$ 

Substitute 4 and 5 in 6, we get,

$$\mu_3 \subseteq \mu_1 \text{ (or) } \mu_3 \subseteq \mu_2 \hspace{1cm} \text{III}.$$ 

From I, II, and III, We have,

$$\mu_1 \subseteq \mu_2 \text{ (or) } \mu_2 \subseteq \mu_1 \subseteq \mu_3 \text{ (or) } \mu_3 \subseteq \mu_2 \subseteq \mu_3 \text{ (or) } \mu_3 \subseteq \mu_2 \text{ (or) } \mu_2 \subseteq \mu_3.$$ 

Hence the union of the three fuzzy sub-trigroups of a group $G$ is a fuzzy sub-trigroups if and only if one is contained in the other.

Similarly, We prove the Theorem for fuzzy sub-quadratic groups and soon fuzzy sub-$n$ groups.

i.e.,

The union of the four fuzzy sub-quadratic groups of a group $G$ is a fuzzy sub-quadratic groups if and only if one is contained in the other.

Similarly for $n$.

The union of the “$n$” fuzzy sub-“$n$” groups of a group $G$ is a fuzzy sub-“$n$” groups if and only if one is contained in the other.

Conclusion:
In this paper, We derive the union of the “$n$” fuzzy sub-“$n$” groups of a group $G$.

For $n = 2, 3, \ldots, n$.

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