Fatigue Life Analysis Of Joint Bar Of Insulated Rail Joint

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Abstract: Rail joints are use for the purpose of joining the two rails. Traditionally, it was done by using bolted rail joint but due to large number of drawbacks and for safety purpose the number of joints in mainline track has been minimized by the widespread use of continuously welded rail (CWR) but for the track circuit there is no alternative for insulated rail joint. Insulated rail joint is use for signaling purpose to allow a railway signaling system to locate trains by maintaining a shorting circuits system. The IRJ is however substantially weaker than the rail and so is subjected to large stresses, causing failure. This paper is concern with to find the fatigue life of joint bar of IRJ and to suggest the proper profile design which will help to improve its fatigue life.

Keywords: Insulated rail joint, joint bar, stresses, fatigue life, wheel rail contact, endpost.

1. Introduction
Failure of insulated rail joint leads to delay in train also sometime may be responsible for train accident. Research indicates that the mean service life of insulated rail joint is only about 20% that of noninsulated joints including CWR with short service life of 12-18 months. Generally the bending stiffness of a rail joint is much smaller than that of the rail. It is about one third. This is a major weakness of the IRJ. The bending stiffness of IRJ depends on several factors such as bending stiffness of joint bars, the tightness of the bolts and how well bars fit in rail ends. Another weakness is the failure of joint bar causing a support discontinuity during passing of train wheel over it. This can cause a vertical acceleration of the moving wheel load and induce a dynamic force and stresses in the track. Combining effects cause a failure readily at the IRJ compared to elsewhere along a continuous rail. Based on various worldwide designs of IRJ’s, the following are the failure modes of IRJ:
i. Bending stress  ii. Thermal stress  iii. Residual stress
iv. Shear stress  v. Von misses stress
vi. Fatigue failure  vii. Broken bolt
viii. Joint bar failure  ix. Delamination of end post
x. Crushed end post and metal flow.
xi. Stresses due to static and Dynamic load
xii. Epoxy layer debonding from rail, joint bar or both

This work focuses on mainly fatigue life of joint bar which is calculated in terms of million gross tons (MGT) which helps to find the different stress developed and also the life expectancy of joint bar. The following three profiles of joint bars have considered for this work.

2. Objective
Objective of this work is to estimate and compare the fatigue life of joint bar with the three different profiles to suggest the best suited design.

3. Component of Insulated Rail Joint
1) Rail  2) Joint bar/fisher plates
3) Insulating End post  4) Nut and Bolts
5) Insulating material (between rail and joint bar).

3.1 Rail: Rail is nothing but the continuous track which is made from carbon steel and joint together by continuous arc welding. In IRJ other part are fastened to rail and then it is welded to other rail on track. 136RE is most common rail type use in India.
3.1.2 Joint bar: Joint bars are two carbon steel plates use to support the IRJ laterally by fastening nut and bolt to IRJ.

3.1.3 Endpost: Endpost is working as an insulating material between two rails for electrical signaling purpose. The Endmost material can be inserted in between two ends of rail perpendicular to longitudinal axis of rail. The thickness of Endpost may vary from 5mm to 10mm. Material use for end post is fiber glass.

3.1.4 Nut and Bolt: Nut and bolt are use for fastening purpose.

4. Methodology
i. Estimation of fatigue life (in terms of MGT) of joint bar using Miner’s law.
ii. Analyze the effect of changing length & cross sectional area on life of Joint bar.

The fatigue life of various joint bar has calculated by considering various lengths in addition to 36 inch joint bar, which is generally used in Indian rail track and various cross section area. Finally, comparing results Joint bar with improved design has selected in order to increase the fatigue life of Joint bar.

5. Specification and Design Consideration

5.1 Specification of Joint bar:
Length of joint bar = 1.219 m (48 inches) 
= 0.9144 m (36 inches) 
= 0.6096 m (24 in)

A) For joint bar profile P1:
Cross Sectional Area, \( A_{J1} = 0.008 \text{ m}^2 \) (6.27 in²)
Moment of Inertia, \( I_{J1} = 5.098 \times 10^{-6} \text{ m}^4 \) (12.25 in⁴)
Topmost distance from neutral axis, \( C_{J1} = 0.0388 \text{ m} \) (1.53 inches)

B) For joint bar profile P2:
Cross Sectional Area, \( A_{J2} = 0.00759 \text{ m}^2 \) (5.89 inch²)
Moment of Inertia, \( I_{J2} = 6.714 \times 10^{-6} \text{ m}^4 \) (16.14 inch⁴)
Topmost distance from neutral axis, \( C_{J2} = 0.0612 \text{ m} \) (2.41 inches)

C) For joint bar profile P3:
Cross Sectional Area, \( A_{J3} = 0.00727 \text{ m}^2 \) (5.64 inch²)
Moment of Inertia, \( I_{J3} = 5.84 \times 10^{-6} \text{ m}^4 \) (14.04 inch⁴)
Topmost distance from neutral axis, \( C_{J3} = 0.0642 \text{ m} \) (2.53 inches)

5.2 Design consideration
Joint bar is a rigid plate of steel which supports a joint bar laterally and use in various lengths and cross sectional area. In India common length of joint bar is 48 inch and 36 inch.

Figure 3. a) 30 mm Joint Bar Profile b) Nut bolt dimension

Figure 4. Joint Bar Hole Spacing.

6. Dynamic Load Calculations

6.1 General Consideration: Insulated joints are subjected to load from two main sources
1) From rail imparting vertical forces to the joint through the wheels.
2) From the thermal loads that occur longitudinally in the joint.

Vertical load = Static load + Dynamic load

The vertical load imparted to the track does not simply come from the static force \( (P_o) \) of wheel pressing down on the rail. Use of static load alone for analysis would only be sound only if it is assumed that rails and wheels are both smooth and free of defects. With actual wheels and rail, however, this is not the case and the presence of defects causes additional dynamic loads \( (P_d) \) to be applied to the track. The vertical load that is actually applied to the rail is super position \( (P_{TOT}) \) and the dynamic load \( (P_d) \).

The major factors that affect the magnitude of dynamic load components are:

i. Speed of train ii. Static wheel load
iii. Wheel diameter iv. Vehicle unstrung mass
v. Condition of the vehicle vi. Condition of track

i. Condition and type of ballast and sub ballast.

A) Dynamic load factor (DLF):

\[
DLF = \left( 1 + \frac{33 \text{V mph}^2}{100 \text{ D}} \right)
\]

\( V_{mph} \) - Train velocity in miles per hour.
\( D \) - Wheel diameter in inches

B) Calculation of dynamic force:

\[
P_d = DLF \times P_o
\]

\( P_{TOT} \) - Total vertical force; \( P_o \) - Static wheel load.

But equation (1) is valid only for continuous track and not for joint in track.
C) Calculation of dynamic load when wheel impact at joint:

When wheel impact a joint, the dynamic load consist of a short time peak (P₀) and delay peak (Pₙ) as shown in fig.

\[ P_D = P_o + \left[ 1 + \frac{\sqrt{K_r \sum a^2 R^2}}{M_u V^2} - 1 \right] \]  

\( a_4 = \frac{1}{230} \)  

\( \Delta = \text{Distance between rail ends at the joint.} \)
\( \alpha_3 = \text{Bending of rail end at joint} \)

\[ a_3 = \frac{P_{32}}{K_v} - \frac{2 \alpha_2}{K_v} \left[ 1 - e^{-\lambda t} \left( \cos \alpha t - \sin \alpha t \right) \right] \]  

This is from semi – infinite beam on elastic foundation theory.

P - Applied wheel load; Q - Joint bar reaction load

\[ t = b \cdot \frac{\Delta}{2} \]  

\[ b \] -Half length of joint bar; D - Distance between rail ends at the joint.

\[ \lambda = 4 \frac{K_v}{4 E I R} \]  

E - Modulus of elasticity of rail material; \( I_R \) - Bending moment of inertia. To calculate joint bar reaction load.

\[ Q = \beta \frac{P}{4 \lambda b} \]  

\( \beta \) - Joint efficiency factor; \( b \) - Half length of joint bar.

Joint efficiency factor is the fraction of the bending moment that the joint bar carry in continuous rail and various between 0.6 to 0.8 for good joint. Theoretical maximum value for joint efficiency factor = 4

\( I_p \) - vertical bending inertia of joint bars
\( I_R \) - vertical bending inertia of rail.

**Table 1. Parameter use in dynamic load calculation.**

<table>
<thead>
<tr>
<th>S. No</th>
<th>Parameter</th>
<th>Value</th>
<th>Value(SI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mu</td>
<td>4.53 lb/s/in</td>
<td>793.32N/m²</td>
</tr>
<tr>
<td>2</td>
<td>Kr</td>
<td>3.3x10^10 lb/in²</td>
<td>57.79 x10^8 N/m²</td>
</tr>
<tr>
<td>3</td>
<td>K_v</td>
<td>3000 psi</td>
<td>20.684 x10^6 N/m²</td>
</tr>
<tr>
<td>4</td>
<td>R</td>
<td>18 in</td>
<td>0.4572 m</td>
</tr>
<tr>
<td>5</td>
<td>R</td>
<td>40 mph</td>
<td>18.22 m/s</td>
</tr>
<tr>
<td>6</td>
<td>E</td>
<td>30 x10^6 psi</td>
<td>2.068 x10^11 N/m³</td>
</tr>
<tr>
<td>7</td>
<td>I_p</td>
<td>94.2 in²</td>
<td>3.92 x10^-3 m³</td>
</tr>
<tr>
<td>8</td>
<td>I_R</td>
<td>3.38 in²</td>
<td>1.34 x10^-3 m³</td>
</tr>
<tr>
<td>9</td>
<td>( \Delta t )</td>
<td>41.5°F</td>
<td>5°C</td>
</tr>
<tr>
<td>10</td>
<td>( \Delta )</td>
<td>0.25 in</td>
<td>0.00635 m</td>
</tr>
<tr>
<td>11</td>
<td>B</td>
<td>18 in</td>
<td>0.4572 m</td>
</tr>
<tr>
<td>12</td>
<td>( \alpha_3 )</td>
<td>11.78 in²</td>
<td>0.00759m²</td>
</tr>
<tr>
<td>13</td>
<td>P₀</td>
<td>1900 lb</td>
<td>84.55x10⁸ N</td>
</tr>
<tr>
<td>14</td>
<td>C_f</td>
<td>2.42 in</td>
<td>0.0614 m</td>
</tr>
<tr>
<td>15</td>
<td>S_o</td>
<td>3200 lb/ in²</td>
<td>220.0632x10⁸ N/m²</td>
</tr>
<tr>
<td>16</td>
<td>S_t</td>
<td>130,000 lb/in²</td>
<td>890.318x10⁸ N/m²</td>
</tr>
<tr>
<td>17</td>
<td>( \alpha_3 )</td>
<td>12.95 in²</td>
<td>0.008354 m</td>
</tr>
<tr>
<td>18</td>
<td>( \alpha_0 )</td>
<td>6.5 x 10⁻²°F</td>
<td>117 x10⁻²°C</td>
</tr>
</tbody>
</table>

\( \alpha_4 \) - Related to deformation of rail ends due to batter. This batter is characterized by its depth and length.

\[ \alpha_4 = \frac{d}{\frac{\Delta}{2}} \]  

\( d \) - Depth of deformation due to rail end batter

D) Calculation for Dynamic load, \( P_d \):

\[ P_d = P_o + \frac{M_u V^2}{R} \left[ 1 + \frac{K_r \sum a^2 R^2}{M_u V^2} - 1 \right] \]  

\[ a = \alpha_4 + \alpha_2 + \alpha_3 + \alpha_4 \]  

\[ \alpha_4 = \frac{1}{2 \lambda} \]  

\[ \alpha_2 = \frac{\alpha_2}{2R} = \frac{0.0635}{2 x 0.4572} = 0.00694 \]  

\[ \alpha_2 = 2 \left[ \frac{P_{32}}{K_v} - \frac{2 \alpha_2}{K_v} \left[ 1 - e^{-\lambda t} \left( \cos \alpha t - \sin \alpha t \right) \right] \right] \]  

\[ t = b \cdot \frac{\Delta}{2} = 0.4572 - \left( \frac{0.0635}{2} \right) = 0.454 m \]
\[ \lambda = \frac{4}{\sqrt[4]{EIR}} \left[ \frac{20.684 \times 10^6}{4 \times 2.068 \times 10^{11} \times 3.92 \times 10^{-5}} \right] = 0.89 \]

\[ \beta_{\text{max}} = 0.7 \, \text{(Considering } \beta = 0.7) \]

\[ Q = \frac{0.7 \times 84.55 \times 10^3}{4 \times 0.89 \times 0.4572} = 36362.64 \text{ N/m} \]

\[ \alpha_1 = 2 \left[ \frac{20.684 \times 10^6}{20.684 \times 10^6} \right] = \frac{2}{2} \left[ \frac{(0.89 \times 0.454) - (0.89 \times 0.454)}{2.785 \times 10^{-3}} \right] = 0.00458 \]

\[ \alpha = \frac{\beta}{\lambda} \]

In the calculation of the vertical loads of the joint, it has been assumed that there was no deformation of the rail end due to rail end batter. Therefore, \( \alpha \) neglected here.

\[ a = \alpha_1 + \alpha_2 + \alpha_3 = 0.049 + 0.00694 + 0.00458 = 0.06 \]

\[ P_d = P_o + \frac{M_o V^2}{R} \left[ 1 + \frac{KPr^2}{Mu V^2} \right] - 1 \]

\[ = 84.55 \times 10^3 + \left( \frac{793.32 \times 10^{22}}{0.4572} \right) \left[ 1 + \left( \frac{57.79 \times 10^6 \times 0.062 \times 0.4572}{793.32 \times 10^{22}} \right) - 1 \right] \]

\[ P_d = 130.29 \times 10^3 \text{ N} \]

Static load, \( P_d = 84.55 \times 10^3 \text{ N} \)

It come from an average wheel load from 150 trains moving over 12 wheel impact load detector (WILD) sites that were samples three times in a 12 month period.

Formula for dynamic load factor (DLF)

\[ DLF = 1 + \frac{33 V_{mph}}{1000} = 1 + \left( \frac{33 \times 18.22}{1000} \right) = 1 + 0.167 = 1.167 \]

\[ P_{TOT} = P_o \times DLF = 19000 \times 1.167 \]

\[ P_{TOT} = 22173 \text{ lb} = 98.66 \times 10^3 \text{ N} \]

### Table 2. Value of Respective Loads

<table>
<thead>
<tr>
<th>S. No</th>
<th>Load</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( P_o )</td>
<td>84.55 \times 10^3 \text{ N}</td>
</tr>
<tr>
<td>2</td>
<td>( P_d )</td>
<td>130.29 \times 10^3 \text{ N}</td>
</tr>
<tr>
<td>3</td>
<td>( P_{TOT} )</td>
<td>98.66 \times 10^3 \text{ N}</td>
</tr>
</tbody>
</table>

#### 7. Fatigue Life Calculation

**7.1 Engineering Estimation of Stresses in joint bars**

For this study, 132 RE rail and its companion 36 inch joint bar shown in fig, have been chosen to illustrate the method. The longitudinal bending moment distribution in a continuous rail is calculated from the equations for an infinite beam on elastic foundation, and is equation.

\[ M_R(x) = \frac{P_d}{4} \left[ \exp(-\lambda x) \right] \left( \cos \lambda x - \sin \lambda x \right) \]  

(1)

Where, \( P_d \) is the dynamic load

\[ \lambda = \frac{4}{\sqrt[4]{EIR}} \]  

(2)

\[ K_R = \text{Rail foundation modulus}; \ E = \text{modulus of elasticity} \]

\[ I_R = \text{Rail bending inertia} = 94.2 \text{ in}^4 \]

The bending moment is maximum at the point of application of the load (\( x = 0 \)) where

\[ M_R(0) = \frac{P_d}{4I} \]

Joint bars are also assumed to behave as beams in bending and are assumed to carry only a fraction of moment in a continuous rail. This fraction is called the joint efficiency factor, \( P \)

\[ \beta_{\text{max}} = \frac{M_J}{M_R} \max = \frac{4I_J}{I_R} \]

(3)

where \( I_J \) and \( I_R \) are the vertical bending inertias for the joint bars and rail respectively, the bending moment carried by the joint bars (\( M_J \)) is assumed to be.

\[ M_J = \beta_m P_d \]

(4)

The maximum bending stress at the top of the joint bar is compressive when the wheel is located at the joint and is equal to:

\[ S_{J+} = \frac{M_J C_J}{I_J} \]

(5)

\( C_J \) = Distance from the joint bar neutral axis to the top of the joint bar as shown in fig.

\( I_J \) = moment of inertia of joint bar pair.

When wheel moves away from the joint location, the moment distribution evolves and the rail is subjected to reverse bending. The maximum reverse bending stress at the top of the joint bar is tensile and occurs when the wheel is at a distance \( x_{rb} \) from the joint, which is determined by differentiating equation the moment at this location is determined by applying equation with \( P = P_{TOT} \) The stress in the joint bar is related to the moment at this location and is equal to the moment at this location and is equal to:

\[ S_{J-} = \frac{\beta_m P_d (x_{rb}) C_J}{I_J} \]

(6)

The fatigue life of joint bars is adversely affected by stress imposed by rail thermal expansion when the ambient temperature is below the rail neutral temperature. The thermal load in rail, \( P_{th} \) is equal to

\[ P_{th} = A_T \Delta T = 117 \times 10^{-7} \text{ f/°C} \]

(7)

\( A_T \) = Rail cross- sectional area;

\( \Delta T \) = Coefficient of thermal expansion

\[ S_{th} = \frac{P_{th}}{X_f} \]

(8)

\( A_J \) = Joint bar cross sectional area; \( \Delta T = 5^\circ \text{C} \) The rail fatigue properties used in the present calculations were derived from experiments conducted by Jensen. The calculations for the fatigue life of Joint bars are carried out using Miner’s law expressed as

\[ N = N_0 \left[ \frac{S_{th} (S_{th} - S_M)}{3S_J S_M} \right]^{1/k} \]

(9)

\( N \) = Fatigue life in cycles; \( N_0 \) = Represents infinite life (in cycles) \( S_{th} \) = Endurance limit; \( S_M \) = Ultimate strength
7.2 Fatigue life calculation for P1:

Given data:
- \( I_R = 3.92 \times 10^{-5} \) m\(^4\); \( I_J = 5.0984 \times 10^{-6} \) m\(^4\)
- \( C_J = 0.0388m; \) \( x_{rb} = 1.143m \)
- \( A_K = 0.008354 m; \) \( A_I = 0.008 \)
- \( a_{th} = 117x 10^{-7} ^\circ C \)
- \( P_a = 84.55 x 10^3 \) N; \( P_d = 130.29 x 10^3 \) N
- \( P_{TOT} = 98.66 x 10^3 \) N; \( \lambda = 0.89 \)
- \( S_I = \frac{M_J}{I_J} \)
- \( M_J = \frac{\beta P_d}{4A} \)
- \( = 0.7x 130.29 x 10^3 \times 408.99 \)
- \( = 25.61 x 10^3 \) N.m

\[ S_{J} = \frac{M_J}{I_J} \]

- \( S_{J} = 6.714 x 10^{-4} \times 10^6 \) N/m²

\[ A)\text{Calculation of } M_R(xrb): \]

\[ M_R(xrb) = \frac{P_{TOT}}{4A} \left[ \exp (-\lambda x) (\cos \lambda x - \sin \lambda x) \right] \]

\[ M_R (45) = 98.66 x 10^3 \]

\[ \frac{4 x 0.89}{\exp (-0.89 x 1.143)[\cos(0.89x1.143) - \sin(0.89x1.143)]} = 27.71 x 10^3 \]

\[ = 27.71 x 10^3 \times 0.1017 x10^5 \times 0.00759 \]

\[ = 75.763 x 10^6 N/m² \]

\[ S_J = \frac{M_J}{I_J} \]

\[ = 75.763 x 10^6 \]

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\[ M_R (45) = 98.66 x 10^3 \]

\[ \frac{4 x 0.89}{\exp (-0.89 x 1.143)[\cos(0.89x1.143) - \sin(0.89x1.143)]} = 27.71 x 10^3 \]

\[ = 27.71 x 10^3 \times 0.1017 x10^5 \times 0.00759 \]

\[ = 75.763 x 10^6 N/m² \]

\[ S_J = \frac{M_J}{I_J} \]

\[ = 75.763 x 10^6 \]
MR(45) = 9.82 x 10^3 N.m
Sb = 0.7x9.82x10^3 x 8.0642 
= 75.56 x 10^6 N/m^2
Pb = A Eb. g. AT = 1.01 x 10^5 N/m^2
Sb = 1.01 x 10^5 
= 0.00277
Sb = 13.89 x 10^6 N/m^2
Smin = Sb + Sb = -267.64 x 10^6 N/m^2
Pth = A Eb. g. th
= 1.01 x 10^5 
= 0.00277
Sth = 13.89 x 10^6 N/m^2
Smax = Sth + Sth = 75.56 x 10^6 N/m^2
Sth = (75.56 x 10^6) + (13.89 x 10^6) = 89.45 x 10^6 N/m^2
Pth = A Eb. g. th
= 1.01 x 10^5 
= 0.00277
Sth = 13.89 x 10^6 N/m^2
N = No = 0.063 x 10^3 cycles
MGT = 26.63

Table 3. Fatigue life of joint with different profiles.

<table>
<thead>
<tr>
<th>S. No</th>
<th>Profile</th>
<th>MGT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P1</td>
<td>271.73</td>
</tr>
<tr>
<td>2</td>
<td>P2</td>
<td>84.45</td>
</tr>
<tr>
<td>3</td>
<td>P3</td>
<td>26.63</td>
</tr>
</tbody>
</table>

8. Effect of Change in Length on Fatigue Life of Joint Bar
From equation of dynamic load calculation it is clear that the length use only in the calculation of dip angle.

\[ \alpha_3 = \left( \frac{P_{12}}{K^2 V^2} \right) \left[ 1 - e^{\frac{-\lambda (\cos \lambda - \sin \lambda)}{\sin \lambda}} \right] \]  
\[\Sigma \alpha = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 \]  
\[P_d = P_o \left[ 1 + \sqrt{\frac{K_r \Sigma \alpha^2 R^2}{M u V^2}} - 1 \right] \]

8.1 Case I:
When L = 1.219 m (48in)
\[ \alpha_1 = 0.049 m; \quad \alpha_2 = 0.00694 m \]
\[ \alpha_3 = \left( \frac{P_{12}}{K^2 V^2} \right) \left[ 1 - e^{\frac{-\lambda (\cos \lambda - \sin \lambda)}{\sin \lambda}} \right] \]
\[ \lambda = \frac{b}{2} = 0.3 - \left( \frac{0.0635}{2} \right) \]
\[Q = \beta \frac{P_0}{4 A_b} \left[ 0.7 x 84.55 x 10^3 \right] \left[ 1 - e^{-\frac{0.89 x 0.4572}{\sin \lambda}} \right] \]
\[\alpha_3 \lambda = \Sigma \alpha \lambda = \alpha_1 + \alpha_2 + \alpha_3 \lambda = \frac{0.06694 + 0.049 + 0.00447}{0.06694 + 0.049 + 0.00447} = 0.06 \]
\[P_d = P_o \left[ 1 + \sqrt{\frac{K_r \Sigma \alpha^2 R^2}{M u V^2}} - 1 \right] \]
\[= 84.55 x 10^3 + \left( \frac{793.32 x 18.22^2}{0.4572} \right) \left( 1 + \frac{57.79 x 10^6 x 0.06694 x 0.4572^2}{793.32 x 18.22^2} \right) \]
\[= 130.29 x 10^3 \]

8.2 Case II:
When L = 0.9144 m (36in)
\[ \alpha_1 = 0.049 m; \quad \alpha_2 = 0.00694 m \]
\[ \alpha_3 = \left( \frac{P_{12}}{K^2 V^2} \right) \left[ 1 - e^{\frac{-\lambda (\cos \lambda - \sin \lambda)}{\sin \lambda}} \right] \]
\[ \lambda = \frac{b}{2} = 0.4572 - \left( \frac{0.0635}{2} \right) \]
\[= 0.454 m \]
\[\alpha_3 \lambda = \Sigma \alpha \lambda = \alpha_1 + \alpha_2 + \alpha_3 \lambda = \frac{0.06694 + 0.049 + 0.00447}{0.06694 + 0.049 + 0.00447} = 0.06 \]
\[P_d = P_o \left[ 1 + \sqrt{\frac{K_r \Sigma \alpha^2 R^2}{M u V^2}} - 1 \right] \]
\[= 84.55 x 10^3 + \left( \frac{793.32 x 18.22^2}{0.4572} \right) \left( 1 + \frac{57.79 x 10^6 x 0.06694 x 0.4572^2}{793.32 x 18.22^2} \right) \]
\[= 130.29 x 10^3 \]

9. Graphical Analysis
Graphical analysis is use her to compare the relation of various input on final result.
Table 5. Profiles with A, I, and C_f

<table>
<thead>
<tr>
<th></th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_0</td>
<td>( 5.09 \times 10^{-6} )</td>
<td>( 7.59 \times 10^{-6} )</td>
<td>( 7.72 \times 10^{-6} )</td>
</tr>
<tr>
<td>I_0</td>
<td>1.53</td>
<td>2.42</td>
<td>2.53</td>
</tr>
<tr>
<td>C_f</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

From above graph, it is clear that
1. As the distance from neutral axis reduces, fatigue life of joint bar is increases.
2. Profile P1 has lowest value of C_f and highest value of MGT.

10. Result

I) Analytical Method:
From table 3, it is clear that profile P1 (MGT = 271.73) has the highest MGT among the other profiles. Also, the value of MGT is depends on following three factors:
1. Cross sectional area
2. Moment of Inertia
3. Neutral axis distance of joint bar from top of rail.

10.2) Graphical Method:
From figures 6, 7 & 8, it is clear that for profile P1:
1. As cross sectional area increases, MGT increases.
2. As moment of inertia increases, in general MGT increases.
3. As distance of neutral axis from top reduces, MGT increases.

11. Conclusion
Proper design of joint bar helps to improve fatigue life which ultimately leads to improve the service life of Insulated Rail Joint. On the basis of analytical and graphical results, it has concluded that fatigue life is mainly depend on the cross-sectional area, moment of inertia and distance of neutral axis from the top. For better service life, cross-sectional area, moment of inertia should be more and distance of neutral axis from top should be less.

12. References


