The Beta-Halfnormal Distribution And Its Properties

AKOMOLAFE.A.A, MARADESA.A

Lecturer, Department of Statistics, Federal University of Technology, P.M.B 704, Akure, Nigeria. +2348039137435, akomolafe01@yahoo.com

Research student, Department of Statistics, Federal University of Technology P.M.B 704, Akure, Nigeria,+2347034361609, maradprime1@gmail.com

ABSTRACT: In this research, we consider certain results characterizing the generalization of Beta and Half Normal distribution through their distribution functions and asymptotic properties. The resulting Beta-HalfNormal Distribution [BHND] was defined and some of its properties like moment generating function, survival rate function, hazard rate function and cumulative distribution function were investigated. The distribution was found to generalize some known distributions thereby providing a great flexibility in modeling symmetric heavy tailed, skewed and bimodal distributions.

Keywords: Beta-HalfNormal distribution, Moment generating function, Hazard rate, Survival rate, Asymptotic properties,

1 Introduction

In recent times, researches have used mixed distribution as an alternative to some distributions, the half-normal distribution is a special case of the folded normal and truncated normal distributions. It was used to model brownian movement and can also be used in the modeling measurement data and lifetime data. Advances in distribution theory indicate that mixed distribution has received appreciable consideration in recent years. Many compound distributions have been derived based on this approach. Let X~N(0,σ^2), then Y = |X| follows half normal distribution. The Half-normal is a fold at the mean of an ordinary normal distribution with mean zero, where σ is the scale parameter.

The beta family of distribution became popular some years back and several works have been done regarding beta compounding with other distribution which include beta-normal(Eugene&Famoye,2002)[2];betaGumbel(Nadarajah&Kotz,2004)[3],betaWeibull(Famoye,lee&Olugbenga,2005)[4], Beta-exponential (Nadarajah&Kotz, 2006) [5]: beta-Rayleigh (Akinsete&Lowe, 2009) [6]:beta-Laplace (Kozubowski&Nadarajah, 2008)[7]; beta-Pareto (Akinsete, Faye & Lee, 2000)[8]; beta-Gamma, beta-F, beta-t, beta-beta, beta-modified weibull, beta-nakagami among others. Let X~N(0,σ^2), then Y = |X| follows half normal distribution, if its probability density function is defined as follows:

f(x) = \frac{\sqrt{2}}{\sigma\sqrt{\pi}} e^{-\frac{x^2}{2\sigma^2}}, x>0

(1)

E(x) = \int_{-\infty}^{\infty} x f(x) dx = \frac{\sqrt{2}}{\sigma\sqrt{\pi}} \int_{-\infty}^{\infty} xe^{-\frac{x^2}{2\sigma^2}} dx ; Let y = \frac{x^2}{2\sigma^2} ;

dy = \frac{2x}{2\sigma^2} \; dx = \sigma^2 \; dy

E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) dx = \frac{\sqrt{2}}{\sigma\sqrt{\pi}} \int_{-\infty}^{\infty} x^2 e^{-\frac{x^2}{2\sigma^2}} dx

(2)

Therefore the mean of half-normal distribution is represented by (2) below;

E(x) = \frac{\sigma\sqrt{2}}{\sqrt{\pi}}

E(x^2) = \frac{2\sigma^2\sqrt{2}}{\sqrt{\pi}}

Since y = \frac{x^2}{2\sigma^2} , then x^2 = 2\sigma^2(y) ; x = \sqrt{2\sigma^2 y} = \sigma\sqrt{2y} .

Since we have established that x = \sqrt{2y} , substitute for x = \sigma\sqrt{2y} in (3).

E(y) = \int_{0}^{\infty} y f(y) e^{-\sigma\sqrt{2y} y} dy = \frac{\sigma^2\sqrt{2}}{\sqrt{\pi}} \int_{0}^{\infty} \sqrt{2y} e^{-\sigma\sqrt{2y} y} dy

(3)

Recall that erf(\infty) = 1, erf(0) = 0

\frac{2\sigma^2}{\sqrt{\pi}} \left( \frac{1}{2\sqrt{\pi}} erf(\sqrt{y}) - e^{-y} \right) = \frac{2\sigma^2}{\sqrt{\pi}} \left( \frac{1}{2\sqrt{\pi}} erf(\infty) - erf(0) \right) = \frac{2\sigma^2}{\sqrt{\pi}} \left( \frac{1}{2\sqrt{\pi}} \right)

Then E(x^2) = \sigma^2

Variance of Half-normal distribution can be obtained as (4).

Var(x) = E(x^2) - (E(x))^2 = \sigma^2 - \left( \frac{\sigma\sqrt{2}}{\sqrt{\pi}} \right)^2 = \sigma^2 - \frac{2\sigma^2}{\pi} = \sigma^2 \left( 1 - \frac{2}{\pi} \right) (4)
2 The proposed Beta-halfnormal Distribution (BHND)

Now by using the logit of beta defined by Jones, the mixture of Beta-Halfnormal distribution can be obtained. Now let X be a random variable from the distribution with parameters and defined by (1) using the logit of Beta defined by Jones as:
\[
g(x) = \frac{1}{B(a,b)} [F(x)]^{a-1} [1 - F(x)]^{b-1} f(x)
\]
(5)

since x~HN(0,σ^2), the we need to obtain the cumulative density of Halfnormal distribution, and it can be obtained as follows:
\[
F_{BHND}(x) = \int_{0}^{x} \frac{\sqrt{2}}{\sigma \sqrt{\pi}} e^{-\frac{x^2}{2\sigma^2}} dx = \int_{0}^{\frac{\sqrt{2} x}{\sigma \sqrt{\pi}}} e^{-t^2} \frac{1}{\sigma \sqrt{\pi}} dt
\]
(6)

The cumulative density can also be obtained in term of error function as follow:
\[
F(x) = \int_{0}^{x} \frac{\sqrt{2}}{\sigma \sqrt{\pi}} e^{-\frac{y^2}{2\sigma^2}} dy (7)
\]

By using the change of variables and let \( y = \frac{x^2}{2\sigma^2} \); \( dy = \frac{x}{\sigma \sqrt{2}} \); \( dx = \sigma \sqrt{2} dy \) by cross multiplication. Now substitute for y and \( dF \) in (7)

\[
F(y; \sigma; a, b) = \int_{0}^{\frac{x^2}{2\sigma^2}} e^{-y^2} \sigma \sqrt{2} dy = \int_{0}^{\sqrt{2} x/\sigma \sqrt{\pi}} e^{-t^2} \frac{1}{\sigma \sqrt{\pi}} dt
\]
(7)

The pdf of Beta-halfnormal can thereby be obtained by applying the logit of Beta function as displayed below
\[
F_{BHND}(y; \sigma; a, b) = \frac{1}{B(a,b)} [F(x)]^{a-1} [1 - F(x)]^{b-1} f(x)
\]
(8)

\[
f_{BHND}(x;a, b, \sigma) = \frac{1}{B(a,b)} [F(x)]^{a-1} [1 - F(x)]^{b-1} f(x)
\]
(9)

The pdf of BHND can be written in it short form when we F(x; \( \sigma; a, b, \sigma \)) is
\[
f_{BHND}(x) = \frac{1}{B(a,b)} Z^{a-1} (1 - Z)^{b-1} Z', \text{ where } Z' = \frac{dZ}{dx}
\]
(10)

2.1 Cumulative Distribution Function (BHND)

\[
F_{BHND}(x) = \int_{0}^{x} \frac{1}{B(a,b)} [\text{erf} \left( \frac{x}{\sigma \sqrt{2}} \right)]^{a-1} [1 - \text{erf} \left( \frac{x}{\sigma \sqrt{2}} \right)]^{b-1} \frac{\sqrt{2}}{\sigma \sqrt{\pi}} e^{-\frac{x^2}{2\sigma^2}} dx
\]
(11)

The incomplete beta function B(z; a, b) can be defined in term of hypergeometric function below:
\[
B(z; a, b) = \sum_{n=0}^{N} \left( \frac{(1-b)(2-b)(n-b)z^n}{n!(a+b)} \right), \text{ where B}(a,b)\text{ is the regularized incomplete beta function}.
\]
(13)

The power series represented by the (13) can be expanded as follow
\[
B(z; a, b) = \sum_{n=0}^{\infty} \frac{(1-b)(2-b)(n-b)z^n}{n!(a+b)}
\]
(14)

F_{BHND}(z) represent the cdf of Beta-Halfnormal Distribution (BHND) as defined in (14).

2.2 The Assymptotic properties

\[
\lim_{x \to \infty} f_{BHND}(x) = \lim_{x \to \infty} \frac{1}{B(a,b)} \left[ \text{erf} \left( \frac{x}{\sigma \sqrt{2}} \right) \right]^{a-1} \left[ 1 - \text{erf} \left( \frac{x}{\sigma \sqrt{2}} \right) \right]^{b-1} f(x)
\]
(11)

\[
\lim_{x \to \infty} \frac{B(a,b)}{B(a,b)} \left[ \text{erf} \left( \frac{x}{\sigma \sqrt{2}} \right) \right]^{a-1} \left[ 1 - \text{erf} \left( \frac{x}{\sigma \sqrt{2}} \right) \right]^{b-1} f(x)
\]
(12)

Limit as \( x \to 0 \)
\[
\lim_{x \to 0} f_{BHND}(x) = \lim_{x \to 0} \frac{1}{B(a,b)} \left[ \text{erf} \left( \frac{x}{\sigma \sqrt{2}} \right) \right]^{a-1} \left[ 1 - \text{erf} \left( \frac{x}{\sigma \sqrt{2}} \right) \right]^{b-1} f(x)
\]
(13)

\[
\lim_{x \to 0} \frac{B(a,b)}{B(a,b)} \left[ \text{erf} \left( \frac{x}{\sigma \sqrt{2}} \right) \right]^{a-1} \left[ 1 - \text{erf} \left( \frac{x}{\sigma \sqrt{2}} \right) \right]^{b-1} f(x)
\]
(14)
If \( x \to 0 \), the pdf \( f_{BHND}(x) \) also tend to zero and as \( x \to \infty \), the pdf \( f_{BHND}(x) \) also tend to zero, then it is an indication that there exist at least on model in pdf of Beta half-normal distribution.

### 2.3 Hazard Rate Function

The hazard rate is of Beta half-normal distribution is expressed as

\[
h(x) = \frac{f_{BHND}(x)}{1-F_{BHND}(x)},
\]

where \( (1-F_{BHND}(x)) \) is the survival rate, and is defined as \( S(x) \), then the survival rate is expressed as:

\[
(1-F_{BHND}(x)) = 1 - \frac{B(2(a,b))}{B(a,b)} = \frac{B(a,b)-B(2(a,b))}{B(a,b)} \]

\[
h(x) = \frac{\theta^{a-1} \epsilon^{a-1}(1-2)^{b-1} x^{b-1}}{b^{a-1} \epsilon^{a-1}(1-2)^{b-1} x^{b-1}} \cdot \frac{X^{b-1}}{B(a,b)-B(2(a,b))}
\]

where \( Z = \text{erf} \left( \frac{\sqrt{2}}{\sqrt{\sigma^2}} \right) \), then

\[
\lim_{x \to \infty} h(x) = \frac{\left( \text{erf} \left( \frac{x}{\sqrt{2}} \right) \right)^{a-1} \left( 1 - \text{erf} \left( \frac{x}{\sqrt{2}} \right) \right)^{b-1} \frac{\sqrt{2}}{\sqrt{\sigma^2}} e^{\frac{x^2}{2\sigma^2}}}{B(a,b) - B \left( \text{erf} \left( \frac{x}{\sqrt{2}} \right); a, b \right)} = 0
\]

\[
\lim_{x \to 0} h(x) = \frac{\left( \text{erf} \left( 0 \right) \right)^{a-1} \left( 1 - \text{erf} \left( 0 \right) \right)^{b-1} \frac{\sqrt{2}}{\sqrt{\sigma^2}} e^{\frac{0^2}{2\sigma^2}}}{B(a,b) - B \left( \text{erf} \left( 0 \right); a, b \right) + \left( \text{erf}(0) \right)^{a-1} \left( 1 - \text{erf}(0) \right)^{b-1} \frac{\sqrt{2}}{\sqrt{\sigma^2}} e^{\frac{0^2}{2\sigma^2}}} = 0
\]

### 2.4 Moment Generating Function

Hosking (1990) describe that when a random variable following a generalized beta generated distribution, such that \( X \sim GBG(a,b,c) \), then \( \mu_i = E \left( X^{i} \right) \), where \( U \sim B(a,b) \), \( c \) is constant and \( F(x) \) is the inverse of CDF of the Halfnormal distribution, since BHND \((a,b,\sigma)\) is a special form when \( c = 1 \). Then the mgf of the proposed distribution \( m(t) \) and the general \( r^{th} \) moment of the beta generated distribution is defined by

\[
\mu_i = \frac{1}{B(a,b)} \int_{0}^{1} \left[ F^{-1}(x) \right]^{r} x^{a-1} \left( 1 - x \right)^{b-1} dx
\]

Cordeiro and de Castro discussed moment generating function of \( \gamma \) for generated beta distribution as:

\[
M(t) = \frac{1}{B(a,b)} \sum_{i=0}^{n} (-1)^{i} (b-1)^{i} \rho(t, ai - 1) \int_{0}^{1} e^{tx} F(x)^{i} f(x) dx
\]

The moment generating function for Beta Halfnormal Distribution is obtained as shown below

\[
M(t) = \frac{1}{B(a,b)} \sum_{i=0}^{n} (-1)^{i} (b-1)^{i} \int_{0}^{1} e^{tx} \left[ \text{erf} \left( \frac{x}{\sqrt{2}} \right) \right]^{a-1} \frac{\sqrt{2}}{\sqrt{\sigma^2}} e^{\frac{x^2}{2\sigma^2}} dx
\]

Let \( a=b=i=1 \), \( M(t) \) become the mgf of parent distribution

### 2.5 Estimation of Parameter

2.5.1 Log-Likelihood Function

Cordeiroet. al [2011] gave the log-likelihood function for \( \theta = (a,b,c,\tau) \), where \( \tau = \sigma \) as

\[
L(\theta) = -n \log C - n \log B(a,b) + \sum_{i=1}^{n} \log F(x_i; \tau) + (a - 1) \sum_{i=1}^{n} \log F(x_i; \tau) + (b - 1) \sum_{i=1}^{n} \log (1 - F(x_i; \tau))
\]

The Generalized Beta distribution reduces to the class of Beta generated distribution when \( c = 1 \). We hereby have \( \theta = (a, b, 1, \tau) \) which can be rewritten as

\[
L(\theta) = -n \log B(a,b) + \sum_{i=1}^{n} \log f(x_i; \tau) + (a - 1) \sum_{i=1}^{n} \log F(x_i; \tau) + (b - 1) \sum_{i=1}^{n} \log (1 - F(x_i; \tau))
\]

\[
= -n \log B(a,b) + \sum_{i=1}^{n} \log \left[ \text{erf} \left( \frac{x_i}{\sqrt{2}} \right) \right] + (b - 1) \sum_{i=1}^{n} \log [1 - \text{erf} \left( \frac{x_i}{\sqrt{2}} \right)]
\]

\[
\frac{\delta L(\theta)}{\delta a} = \frac{n u(a) + n(a+b) r(a+b)}{r(a)} + \sum_{i=1}^{n} \log \left[ \text{erf} \left( \frac{x_i}{\sqrt{2}} \right) \right]
\]

\[
\frac{\delta L(\theta)}{\delta b} = \frac{n u(a) + n(a+b) r(a+b)}{r(a)} + \sum_{i=1}^{n} \log [1 - \text{erf} \left( \frac{x_i}{\sqrt{2}} \right)]
\]

\[
\frac{\delta L(\theta)}{\delta \tau} = \frac{n u(a) + n(a+b) r(a+b)}{r(a)} + \sum_{i=1}^{n} \log \left[ \text{erf} \left( \frac{x_i}{\sqrt{2}} \right) \right] + (b - 1) \sum_{i=1}^{n} \log [1 - \text{erf} \left( \frac{x_i}{\sqrt{2}} \right)]
\]

Since \( \frac{\delta}{\delta \tau} \log \left[ \text{erf} \left( \frac{x_i}{\sqrt{2}} \right) \right] \) can be obtained as follows
When \( \text{erf}(x) \) is expressed in terms of confluent hypergeometric function of the first kind
\[
\frac{2x}{\sqrt{\pi}} M \left( \frac{1}{2}, \frac{3}{2}, -x^2 \right) = \frac{2x}{\sqrt{\pi}} e^{-x^2} M \left( 1, \frac{3}{2}, x^2 \right)
\]
Therefore \( \frac{d^n}{dx^n} \text{erf}(x) = (-1)^n \frac{2}{\sqrt{\pi}} H_{n-1}(x) e^{-x^2} \);

where \( H_{n-1}(x) \) is Hermite polynomial
\[
\text{erf}(x) = \frac{2}{\sqrt{\pi}} \gamma \left( \frac{1}{2}, x^2 \right), \quad \text{where} \quad \gamma \left( \frac{1}{2}, x^2 \right)
\]
\( \text{erf}(x) \) could be viewed as incomplete gamma function, it can therefore be expressed by the Maclaurin series in (19).

\[
\frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{n!(2n+1)} (19)
\]

At \( x = 0, \text{erf}(x) = \frac{2}{\sqrt{\pi}} \left( x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \right) \)
At \( x = \infty, \text{erf}(x) = 1 - e^{-x^2} \left( \frac{1}{2x^2} + \frac{1.3}{(2x^2)^2} + \frac{1.35}{(2x^2)^3} + \cdots \right) \)

By following (19), \( \frac{\partial}{\partial \sigma} \log \text{erf} \left( \frac{x}{\sigma \sqrt{2}} \right) \) can be viewed as complete gamma function, \( \text{erf} \left( \frac{x}{\sigma \sqrt{2}} \right) \) or \( (x > 0) \) and \( \text{erf} \left( \frac{x}{\sigma \sqrt{2}} \right) \) \( \in R \)

By using (19), the series expansion at \( \sigma = 0 \) represented by (20).

\[
e^{-\frac{x^2}{2\sigma^2}} \left( 1 - \frac{1}{\sqrt{\pi}} + \frac{0(\sigma^2)}{1} \right) \left( - \frac{1}{2n} \text{arg}(\sigma) \text{arg}(-x) \right) \left( \frac{x^2}{2n} \right)^{2n} + \frac{1}{x} (21)
\]

By using (19), the series expansion as at \( \sigma = \infty \) represented by (21).

\[
= \log \left( \frac{x \sqrt{\pi}}{\sigma} \right) + 0 \left( \frac{1}{\sigma} \right)^2 (21)
\]

Therefore \( \frac{\partial}{\partial \sigma} \log \text{erf} \left( \frac{x}{\sigma \sqrt{2}} \right) = \frac{x}{2\sigma^2 \sqrt{2}} e^{-x^2} \text{erf} \left( \frac{x}{\sigma \sqrt{2}} \right) \) \( \in R \)

Also, \( \frac{\partial}{\partial \sigma} \log \left[ 1 - \text{erf} \left( \frac{x}{\sigma \sqrt{2}} \right) \right] \) can also be obtained as follows

\[
\text{Real root} \sigma \neq 0, x = 0
\]

Domain \( (\sigma R; x \neq 0) \) and \( \text{erf} \left( \frac{x}{\sigma \sqrt{2}} \right) \) \( \in R \), the root for the variable \( x; \sigma = 0 \)

Series at \( \sigma = 0 \);

\[
\log \left( e^{-\frac{x^2}{2\sigma^2}} + \frac{1}{x} \left[ \frac{1}{2n} \text{arg}(\sigma) \text{arg}(-x) \right] \left( \frac{x^2}{2n} \right)^{2n} \right) \]

Series expansion at \( \sigma = \infty \)

\[
= -\frac{x}{\sqrt{\pi \sigma}} + \frac{x^2}{3! \sqrt{\pi \sigma^3}} + \frac{x^4}{3 \pi^2 \sigma^5} + \frac{0(\sigma^2)}{1} \frac{x^6}{5! \sqrt{\pi \sigma^7}}
\]

therefore \( \frac{\partial}{\partial \sigma} \log \left[ 1 - \text{erf} \left( \frac{x}{\sigma \sqrt{2}} \right) \right] = \frac{x}{\sigma^2 \sqrt{2}} e^{-x^2} \text{erf} \left( \frac{x}{\sigma \sqrt{2}} \right) \) \( \in R \)

Substitute (21 and 22) in (18)

\[
\frac{\partial L(\theta)}{\partial \sigma} = \sum_{i=1}^{n} \frac{\partial}{\partial \sigma} \log \left[ \frac{\sqrt{2}}{\sigma \sqrt{\pi}} e^{-\frac{x_i^2}{2\sigma^2}} \right] + (a - 1) \sum_{i=1}^{n} \frac{\partial}{\partial \sigma} \log \left[ \text{erf} \left( \frac{x_i}{\sigma \sqrt{2}} \right) \right] + (b - 1) \sum_{i=1}^{n} \frac{\partial}{\partial \sigma} \log \left[ 1 - \text{erf} \left( \frac{x_i}{\sigma \sqrt{2}} \right) \right]
\]

\[
= \sum_{i=1}^{n} \left[ \frac{1}{\sigma \sqrt{2}} + \frac{x_i^2}{\sigma^2} \right] + (a - 1) \sum_{i=1}^{n} \frac{x_i e^{-\frac{x_i^2}{2\sigma^2}}}{\sigma^2 \text{erf} \left( \frac{x_i}{\sigma \sqrt{2}} \right)} + (b - 1) \sum_{i=1}^{n} \frac{x_i e^{-\frac{x_i^2}{2\sigma^2}}}{\sigma^2 \text{erf} \left( \frac{x_i}{\sigma \sqrt{2}} \right)} = 0 (23)
\]

The equation 16, 17 and 23 can be solved using Newton Raphson Algorithm to obtain the estimate of \( \hat{a}, \hat{b} \) and \( \hat{\sigma} \), the MLE of \( (a, b, \sigma) \). To test the necessary hypothesis about the significance of the model parameters and the construction of confidence interval, we need to obtain the second derivative of equation (16, 17 and 23) with respect to the parameters of interest with a view to constructing Fisher’s information matrix.

2.5.2 Maximum A Posteriori Estimation (MAPE)

When we consider \( \sigma \) as a random variable, the Bayes convert our belief about the parameter \( \sigma \) of Halfnormal distribution (before seeing data) into posterior probability, \( P(\sigma | X) \), by using the likelihood function \( P(X | \sigma) \). The maximum a-posteriori (MAP) estimate is defined as:

\[
\sigma = \arg \max P(\sigma | X) = \arg \max \frac{P(X | \sigma) P(\sigma)}{P(X)}
\]

The MAP gives the researcher the flexibility of injecting his prior belief about the parameter estimate into the new estimate.

\[
\sigma = \arg \max \prod_{i=1}^{n} P(X | \sigma) P(\sigma) = \arg \max \prod_{i=1}^{n} \log P(X | \sigma) + \log P(\sigma)
\]

\[
\log L = n \log \sqrt{2} - n \log \sigma - \frac{\Sigma x^2}{2\sigma^2} + \log B(a, b)^{-1} + (a - 1) \log \sigma + (b - 1) \log (1 - \sigma)
\]

\[
\frac{\partial \log L}{\partial \sigma} = -n + \frac{\Sigma x^2}{\sigma^2} + \frac{(a - 1)(b - 1)}{\sigma(1 - \sigma)} = 0 (24)
\]

\[
\frac{\partial \log L}{\partial a} = \log \sigma + \frac{\partial}{\partial a} \log B(a, b)^{-1} = 0 (25)
\]

\[
\frac{\partial \log L}{\partial b} = \log (1 - \sigma) + \frac{\partial}{\partial b} \log B(a, b)^{-1} = 0 (26)
\]

By solving equation (24, 25 and 26) the estimate of \( (a, b, \sigma) \) can be obtained, and significant for the parameter of the model can be obtained via log likelihood, AIC or BIC.
Let \( -n + \frac{\sum x^2}{\sigma^2} = 0 \), then \( n = \frac{\sum x^2}{\sigma^2} \), by crossmultiplication, we will obtain the estimate of \( \sigma \) From the Half-normal distribution as \( \hat{\sigma} = \frac{\sqrt{\sum x^2}}{n} \).

The equation derived above can also be solved using iteration method [Newton Rapson] to obtain parameter estimates using maximum likelihood estimation technique. Taking the second derivatives of the equations 24, 25, 26 with respect to the parameter, it is possible to derive the interval estimate and hypothesis tests on the model parameters. This will be shown in our further research.

3 Conclusions

By compounding two or more probability distributions, we get the corresponding hybrid distribution with increased number of parameters which is believed to give the newly compounded distribution more flexibility, consistency, stability, sufficiency uniqueness and wider applicability as compare to its parent distribution. Therefore, the hybrid distribution Beta-halfnormal Mixture distribution is said to have vast applicability extending beyond modeling simple Brownian movement but can be used to model statistical behavior of stochastic processes such as the growth of tumors’ cells in an oncology’s study of benign transmogrifying to malignant tumor, studying the tumors’ cells in an oncology’s study of benign transmogrifying to malignant tumor, studying the consumers’ buying behavior and in complex epidemiological studies because of its increased number of parameters which give the hybrid distribution more flexibility to model many stochastic phenomena.

References


Author’s Profile

First Author

Akomolafe Abayomi Ayodele received a Bsc in Statistics (University of Ilorin, Nigeria), Msc and Phd in Statistics (University of Ibadan, Nigeria) in 2004 and 2011 respectively. Since then he has been a university lecturer that specializes in statistical inference, Stochastic process and applied sample survey. Having acquired more than 15years of university teaching experience, he is presently a senior lecturer at the Federal University of Technology Akure (FUTA), Nigeria.

Second Author

Maradesa, Adeleke received Bachelor of Technology (Btech) in Statistics (Federal University of Technology Akure, Nigeria) and National diploma in Statistics (Federal School of Statistics Ibadan, Nigeria) in 2016 and 2012 respectively. He received adequate training in General Household Survey (GHS), Multiple Indicator Cluster Survey, Farm Survey and Retail price at National Bureau of Statistics, Nigeria (NBS) between 2010 and 2012. He is currently a research student at Federal university of technology Akure Nigeria. His areas of research interest are: Mathematical Statistics, Stochastic Process, and Probability Distribution Theory (Hybrid Distribution) with application to epidemiological studies, genetic modeling and microarray.