Design Of FIR Low Pass Filter Using Particle Swarm Optimization Algorithm

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Abstract: Digital filter play a significant role in the field of digital signal processing. In this paper, a linear phase low pass FIR filter has been analyzed by an approach optimal design method using of particle swarm optimization (PSO) algorithm. The FIR filter involves multi-parameter optimization. Different optimization techniques can be utilized to minimize the coefficients of FIR filter by minimization or maximization of fitness function. The concept of optimization is minimizing the maximum errors between desired and actual response. The main target behind the designing of this improved FIR filter is to approximate the ideal filters on the request of a given designing specifications. The inertia weight has been modified for the PSO to enhance its searching capability for obtain global optimum solution. In the process of designing a FIR low pass filter, specification is realized using PSO algorithm generates a set of filter coefficients and tries to meet the ideal frequency characteristic, feasible pass band and stop band frequencies are required. A comparison of simulation results reveals the optimization efficiency of the algorithm over the prevailing optimization technique for solution, non-differential, highly non-linear and contained FIR filter design problem. Simulation study supports that the proposed algorithm is accurate and has fast convergence speed.

Keywords: Finite impulse response (FIR) filter, Low pass (LP) filter, Evolutionary algorithm, PSO,

1. Introduction
The most important part of DSP is filters which takes a digital input and gives a digital output. The most common form of signal processing used to remove the frequencies in selective part and to improve magnitude, phase or group delay of the spectrum of signal. FIR filter is attractive choices because of the ease in design and good in stability. Digital filter have wide variety of application like signal processing, control system, telecommunication, in various audio and video processing and in defense equipment etc. Digital filter are better than analog filter due to its wide range of application and better performance like small in size, reliable, gives higher accuracy. Digital filter are two types: finite impulse response (FIR) and infinite impulse response (IIR). Traditionally there are many well known method for digital filter design, such as window method, frequency sampling method, etc. The windowing method consists of window function as Butterworth, Chebyshev, Kaiser etc. for FIR filters and which further transformed to digital low pass , high pass, band pass filters. The window method for digital filter design is fast, convenient, robust but generally suboptimal. The main objective function for the design of optimal digital filters involving accurate control of various parameters of frequency spectrum and highly non-uniform, nonlinear, non-differentiable and multimodal in nature. Chebyshev FIR digital filter design is most frequently used method for exact filter design. Most efficient method for the design of exact linear phase weighted Chebyshev. FIR digital filter is the one based on the Remez-exchange algorithm proposed by parks and McClellan but due to its computation complexity and high pass band ripples. Such filter has great error in filter design and need optimization. But classical optimization methods cannot optimize as such objectives function and cannot converge to the global minimum solution. Particle swarm optimization technique have been recognized for the design of optimal digital filters with superior control of parameters and the highest stop band attenuation, various heuristics and stochastic optimization method have been developed, which have proved themselves quite efficient and for the design of digital filter like GA, Differential evolution etc. Here in this paper we are exploring the benefits of design FIR filter using a stochastic technique known as Practical swarm optimization algorithm. In many aspects PSO its self proves to be far more efficient than previously discussed techniques. The Practical swarm optimization is an evolutionary optimization technique developed by Eberhartet.al. The merits of PSO lie in its simplicity to implement as well as its convergence can control via few parameters. Numerous works has been done in order to explored flexibility of FIR filter design provided by PSO and The different alteration of PSO

2. Problem formulation
Filter is classified as non-recursive type filter which means it requires no feedback. The lacks of feedback ensure that the impulse response will be finite in duration. The main advantages of FIR filter is its linear phase frequency response which can easily extract. That is why almost all design method described in the literature deal with filter wit this property. FIR filter structure is always stable, and can design to have a linear phase response. The impulse response h(n) are to be determine in the design process and the values of h(n) will be determine the type of the filter e.g. low pass, high pass, pass band and band stop etc. there are three broad criteria on which filter designed is based , namely the filter should provide as little distortion as possible to the signal; flat pass band; exhibit attenuation characteristics with the highest stop band possible. A digital filter is characterized by,

\[ H(z) = \sum_{n=0}^{N} h(n)z^{-n}, \quad n = 0, 1, \ldots, N \]  

(1)

Where N is the order of the filter which has (N+1) number of coefficient of h(n) as the filter impulse response
Let us assume,
\[ A(\omega) = \sum_{n=-\infty}^{\infty} h(n) \cos \omega n \]  
(2)

Such that \( A(\omega) \) is the amplitude of approximate filter response with real value function of \( \omega \). The above equation results as:
\[ H(e^{j\omega}) = A(\omega)e^{-j\omega} \]  
(3)

The particle vector is distributed in a D dimensional search space, where D =N+1 fir the case of Nth order FIR filter. Suppose our desired frequency response of FIR digital filter as
\[ H_d(e^{-j\omega}) = F_d(\omega)e^{-j\omega} \]  
(4)

where \[ F_d(\omega) = \begin{cases} 1 & \text{for } 0 \leq \omega \leq \omega_p \\ 0 & \text{for } \omega_p < \omega \leq \omega_p \end{cases} \]  
(5)

Here \( D(\omega) \) is the amplitude of filter of desired frequency response vector. \( \omega_p \) and \( \omega_s \) is the pass band and stop band frequencies. Here frequency is sampled in [0, \( \pi \)] with \( m \) sampling points; the position of each particles vector in D dimensional search space represents the same coefficients \( h(n) \) of the transfer function. In the present paper, error fitness function gives by (3) has been adopted in order to achieve minimum ripples in pass band and stop band optimum transition width. By using (3) the PSO filter design technique found better results are obtained over other optimization techniques.
\[ E(\omega) = G(\omega)[H(\omega) - F_d(\omega)] \]  
(6)

In the eq. (3), \( G(\omega) \) the weighting functions. The transition band have selected depend on pass edge and stop band edge frequencies. The error function given in (3) is to be decreased using evolutionary algorithm PSO.
\[ G(\omega) = \begin{cases} \frac{\omega - \omega_p}{\omega_s - \omega_p} & \omega \in [\omega_p, \omega_s] \\ 1 & \omega \in [0, \omega_p] \end{cases} \]  
(7)

PSO tries to minimize this error fitness and hence optimizes the filter performance. The fitness function combined with proposed filter design technique result in achieving minimum pass band and stop band ripples:
\[ \text{Fit1} = \max_{n \in [0, \pi]} \{ |E(\omega)| \} \]  
(8)

This minimize the error function by using fitness function describe in the eq. (8).

3. Particle Swarm optimization algorithm

PSO is a population based optimization algorithm put forward originally by kennedy (a social psychologist) and Eberhart (an electrical engineer)[10]. It is inspired by the observation of social behavior of bird flocking and fish schooling. PSO is simple fast, require less storage and easy in programming. PSO is implicit parallelism which can easily handle with the non-differential objective functions. It is based on the natural process in which swarm of particles to share individual knowledge. Bird flocking and fish schooling optimize a certain objective function’s algorithm used number of particle vectors moving around in the solution space searching for the optimist solution. In PSO every particle remembers its best solution (\( P_{\text{best}} \)) as well as the group’s best solution (\( G_{\text{best}} \)). It is worked on the concept of “constructive cooperation” between particle so that it is easily able to solve multidimensional optimization problem. Each particle tries to modify its position using the following information: The distance between current position and the gbest. The distance between current position and the pbest. Depending on the order of the filter to be designed every particle consists of components or sub-string. In PSO each flying particle adjusts their position and velocity based on their personal experiences in which position and velocity of it particles are represented as:
\[ X_i = (X_{i1}, X_{i2}, \ldots \ldots \ldots , X_{id}) \]  
(3)

\[ V_i = (V_{i1}, V_{i2}, \ldots \ldots \ldots , V_{id}) \]  
(4)

Mathematically, velocities of each particle vector are modified according to the following equation:
\[ v_{i+1} = c_1r_1v_{i+1} + c_2r_2(p_{\text{best}} - X_i) + c_3r_3(g_{\text{best}} - X_i) \]  
(5)

\( v_i \) is velocity of vector \( i \)th at \( k \)th iteration, \( w \) is the weighting factor.
\[ p_{\text{best}} = (p_{\text{best1}}, p_{\text{best2}}, \ldots \ldots \ldots p_{\text{bestN}}) \]  

The error function given in (3) is to be used over other optimization techniques. In the equation (5), the first term is the previous velocity of the particle vector. The second and third terms are used to change the velocity of particle vector. With second and third terms particle keep on “flying” in the same direction until it hits the boundary. It is inertia which tries to explore new areas. In general, initial solution is usually far from global optimum and hence the larger inertia weight may be proved to be beneficial. The larger inertia weight enable the PSO to explore globally and small inertia weight the enable the PSO to explore locally. PSO Parameter \( w \) is the inertia weight in the nth iteration. For better convergence linearly damped inertia is preferred. Usually parameter \( w \) is linearly decreased according to the following equation:
\[ w = w_{\text{max}} - \frac{k\text{iter}}{\text{iter}_{\text{max}}} \]  
(7)

The parameter \( w_{\text{max}} \) is initial and final weight respectively.[3] Each particle moves towards its best and gbest locations and performance of each particle is measured according to a fitness function. In equation (3) second and third term are used for velocity changing of particle vector. Without second and third term particle keep on “flying” in same direction until hits the boundary. The parameter \( w \) is a kind of inertia and tries to explore new areas. Here vector is term \( m \) string of real filter coefficients (\( N/2+1 \)) for even and odd order FIR filter. The searching point in the solution space may be modified by eq:
\[ X_{i+1} = X_i + \mu \]  
(6)

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\[ Cfa = \frac{4}{|2-\phi-\sqrt{4-4\phi}|} \]  
(8)

where \( \phi = c1 + c2 > 4 \)

The velocity update equation is specified in equation (3) uses a constriction factor method and thus convergence is independent of the iteration cycles. For each iteration of the algorithm particle are moves according to the equation (5) and (6) and keeping in the bound of maximum velocity specified in iteration process.
Table 1: Basic steps of PSO algorithm

**Step 1.** Initialize the swarm size, no of sample taken, filter order, sampling frequency, maximum iteration cycle (itmax). Specify the fixed parameters used for the design of low pass linear phase fir filter. This include pass band edge frequency, \( \omega_p = 0.25 \); stop band edge frequency, \( \omega_s = 0.5 \); pass band ripple, \( \delta_p = 0.1 \); stop band ripple, \( \delta_s = 0.01 \); order of the filter \( n = 20 \).

**Step 2.** Initialize the population of particles with \( N = 30 \); number of dimension (filter coefficient), \( D = 11 \); minimum and maximum value for the population of filter coefficients \( h_{\text{min}} = -2 \); \( h_{\text{max}} = 2 \); velocity range, \( v_{\text{min}} = 0.01 \); \( v_{\text{max}} = 0.1 \); number of samples taken = 128; sampling frequency, \( f_s = 1 \)Hz; maximum no of iteration, \( \text{itmax} = 250 \).

**Step 3.** Generate the initial particle vector using above defined parameter and calculate initial error fitness value for the whole population using

**Step 4.** Calculate the minimum fitness value from error fitness vector and compute individual best and group best values from whole population.

**Step 5.** Update the position and velocity according to the equation (3) and eq. (4), taken them now initial as the initial particle vectors; also calculate the error fitness value from these updated parameters and pbest and gbest accordingly.

**Step 6.** If value of the vector pbest and gbest calculated in step 5 are better than those calculated in step 6, then replace the vectors and no change if otherwise.

**Step 7.** Iterate continuously from step 4 to step 6 until convergence criteria is met.

4. Simulation result and discussion

The Simulation for a 20th order FIR LP filter is carried out using MATLAB 2010a. TABLE I bellow shows the parameters taken for PSO and General. The actual filter design used all the parameter of ideal filter defined here as:

\[
h_d = \begin{cases} 
1 & \text{for } 0 \leq \omega \leq 0.225 \\
0 & \text{for } 0.275 \leq \omega \leq 0.5 
\end{cases}
\]

Where \( \omega_c = 0.225 \), represents the cut off frequency of the filter. From this the pass band and stop band edge frequencies of the filter are calculated as:

\[
\omega_p = \omega_c - 0.25 = 0.25 \\
\omega_s = \omega_c + 0.25 = 0.3
\]

**Table 2. Different parameters used in PSO**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>PSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population size</td>
<td>100</td>
</tr>
<tr>
<td>Iteration</td>
<td>500</td>
</tr>
<tr>
<td>( h_{\text{max}} )</td>
<td>2</td>
</tr>
<tr>
<td>( h_{\text{min}} )</td>
<td>-2</td>
</tr>
<tr>
<td>( v_{\text{max}} )</td>
<td>0</td>
</tr>
<tr>
<td>( v_{\text{min}} )</td>
<td>0.1</td>
</tr>
<tr>
<td>C1</td>
<td>2</td>
</tr>
<tr>
<td>C2</td>
<td>2</td>
</tr>
</tbody>
</table>

**Table 3. Optimized filter coefficients obtained by PSO algorithm**

<table>
<thead>
<tr>
<th>( h(n) )</th>
<th>Filter coefficient using PSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h(1) )=h(21)</td>
<td>0.012646021396040</td>
</tr>
<tr>
<td>( h(2) )=h(20)</td>
<td>0.003821625780231</td>
</tr>
<tr>
<td>( h(3) )=h(19)</td>
<td>-0.018023735457979</td>
</tr>
<tr>
<td>( h(4) )=h(18)</td>
<td>-0.022146854865091</td>
</tr>
<tr>
<td>( h(5) )=h(17)</td>
<td>0.035549883427456</td>
</tr>
<tr>
<td>( h(6) )=h(16)</td>
<td>0.042950659046112</td>
</tr>
<tr>
<td>( h(7) )=h(15)</td>
<td>-0.0546786901632457</td>
</tr>
<tr>
<td>( h(8) )=h(14)</td>
<td>0.0417765035521895</td>
</tr>
<tr>
<td>( h(9) )=h(13)</td>
<td>0.0568324511080031</td>
</tr>
<tr>
<td>( h(10) )=h(12)</td>
<td>0.0542034870356813</td>
</tr>
</tbody>
</table>

The design includes 21 coefficients; sampling frequency is 500 Hz, 128 numbers of samples. After running 500 iterations, the best average values of algorithm are recorded. Here normalized pass band and stop band frequency are 0.08 and 0.3 and transition width is 0.01.

**Fig.1 Convergence curve of the FIR low Pass filter of order 20**

**Fig.2 Convergence curve of the FIR low Pass filter of order 20**

5. Figures and Tables

V. Conclusion

The worked done in this paper based on using of algorithm particle swarm optimization based on non-linear multidimensional FIR filter and a easy comparison is made between PSO and General technique for the same kind of filter. Simulation result is clearly indicates that filter design using PSO gives better result in term of maximum pass band and stop band ripples with attenuation and transition width. PSO has the best capability to converge to global optimum narrow transition width.
References


Author Profile

Author 1 received the B.Tech in Electroins and communication form BK Birla , Pilani. and M.tech degrees in Electronics and communication l Engineering from Guru Jambheshwar University Science and Technology , Hisar. During 2015-2017, she stayed in Research Laboratory of Digital signal processing at GJUS&T, Hisar.