Modeling And Forecasting Volatility Of Price Inflation In Ethiopia Using GARCH Family Models

Dereje Demeke Danoro

Dereje Demeke, Hossana, Ethiopia, P.O.Box, 302, Ethiopia
asmebest08@gmail.com

Abstract: Inflation and its volatility is one of the serious macro-economic problems in every countries economy. Inflation in Ethiopia is not immune from volatility problem nowadays and it was vital to model and forecast it. Therefore, this study aimed at modelling and forecasting price inflation volatility in Ethiopia using GARCH family models for the general inflation data, which spans from 1995 to2016. The Lagrangian Multiplier (ARCH -LM) test statistic was used for testing the existence of ARCH effect in the residuals of conditional mean or ARMA (1,1) model and it confirmed the existence of ARCH effect in the log-return series of the price inflation in Ethiopia. This indicates that price inflation in Ethiopia is suffered from volatility problems and applying GARCH family models is relevant and necessary. To model and forecast the price inflation volatility, ARMA (1,1)-GARCH (1,1) was selected as an appropriate model among EGARCH (1,1) and GARCH (1,1) models with GED, normal and t-distributional assumption for residuals. To select an appropriate model, forecasting error measure statistics such as: MAE (Mean Absolute Error), RMSE(Root Mean Square Error)and Uthail's inequality coefficient were used in addition to well-known information criteria's such as: AIC (Akeike Information Criteria) and BIC (Byesian Information Criteria).Moreover, macro-economic variables such as: Broad Money Supply, Exchange Rate and Lending Interest Rate have direct contribution for the price inflation volatility in Ethiopia except Deposit Interest Rate and GDP(Gross Domestic Product). The finding of this study also clearly showed that last shock and volatility had significant contribution to price inflation volatility. Finally, the price inflation volatility was forecasted using ARMA (1,1)-GARCH (1,1) model with GED distributional assumption. The forecast showed the existence of fluctuation of variance which is declining at the end of the study period. This study suggested that, to come up with stable price inflation volatility in Ethiopia, the government as well as concerning bodies must pay great effort to control macro-economic factors of inflation volatility.

Keywords: ARMA; EGARCH; GARCH; Forecasting; Inflation

1. Introduction

Macro-economic problems are serious problems in every countries economy. Among those problems, high inflation rate and its volatility can be mentioned in the forefront. Therefore, reducing high inflation rate and its volatility must be the objective of every countries economic agenda. According to [2], in the earlier periods, inflation in Ethiopia was not a serious problem and had no relationship with agriculture and food supply shock. Major inflationary problems were happening during conflict, war and drought times. However, the recent inflation trajectory seems a new phenomenon. Consistently, according to [2], general inflation rate in Ethiopia is beyond the break-even point and instead of stimulating economic growth; inflationary pressure in Ethiopia seems to be on the verge of distorting the allocation of resources and is likely to be a deterrent to undertaking productive investments. Recent price inflation in Ethiopia is suffered from volatility problem and requires empirical attentions of modeling volatility. Many researchers used VAR, VECM and other econometric approaches to model the Ethiopian price inflation. Regarding this, [3] used monthly data and estimated error correction models to identify the relative importance of several factors contributing to overall inflation in Ethiopia. [4] Provided an assessment of the main derivatives of inflation in Ethiopia and Kenya using VECM for the consumer price index in each country. However, price inflation volatility in Ethiopia is highly fluctuating through time and it needs application of random variance (volatility) models. Therefore, in this study, an appropriate model was selected from GARCH family models. Finally, forecasting and modeling objectives were achieved for the return series of price inflation volatility to answer the following research questions.

1) Is there price inflation volatility in Ethiopia and any model among GARCH family models, which is appropriate for modeling price inflation volatility?
2) Which macro-economic factors are significantly affecting the price inflation volatility?
3) Which model appropriately forecast the price inflation volatility in Ethiopia?

2. DATA AND METHODOLOGY

2.1. Data and Variables of the Study

The data relevant to this study has taken from secondary source recorded data by the Central Statistical Agency of Ethiopia (CSA) for monthly price inflation. For variables, gross domestic product (GDP), money supply, interest rate and exchange rate, the data was collected from national bank of Ethiopia (NBE). The data for the study starts from January 1995 and ends at December 2016. Having this data, this study focused on the following variables.

Dependent variable: Monthly price inflation rate in Ethiopia.

Independent variables: Money Supply, Gross Domestic Product (GDP), Lending Interest Rate, Deposit Interest Rate and Exchange Rate are explanatory variables.

2.2. Methodology

2.2.1. The ARMA (Autoregressive and Moving Average) model

The ARMA model is preferable to the MA or AR models it needs few parameters. Thus, the AR and MA models are often generalized to the most parsimony us model such as ARMA [5]. A mixed autoregressive moving average model with (p) autoregressive terms and (q) moving average terms can be written as:

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + ,..., + \phi_p + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + ... + \theta_q \varepsilon_{t-q}$$

$$\phi(z) y_t = \theta(z) = \varepsilon_t$$

Where $\phi(z)$ and $\theta(z)$ are characteristic equations of the AR and MA processes [17].

2.2.2. Diagnosis test of $ARMA(p,q)$ model

2.2.2.1. Test for the serial correlation of the conditional mean model

One of the basic assumptions of conditional mean model is the residuals must be serially uncorrelated. The Q-statistic is often used as a test of whether the series is white noise. The Q-statistic at lag order(m)is a test statistic for the null hypothesis that there is no autocorrelation up to order m and is computed as:

$$Q = T(T+2)\sum_{j=1}^{m} \frac{\hat{\rho}_j^2}{T-j}$$

Where $\hat{\rho}_j$ is the j[th] lag autocorrelation and T is the number of observations.

Hypothesis:

$H_0 : \rho_1 = \rho_2 = ... = \rho_m$

$H_1 : \rho_m \neq 0$

We reject H_0 (there is no serial autocorrelations up to lag m) if $Q > \chi_\alpha^2 (n-k)$, [7]

2.2.1.2. Jarque-Bera test of normality

For the residuals of the conditional mean models to be adequate, the normality must be tested in addition to serial correlation test. This is applied in this study using the usual normality test statistic called Jarque-Bera.

$$\hat{b}_1 = \hat{\mu}_3 / \hat{\alpha}_3 = \frac{\frac{1}{T}\sum_{i=1}^{T}(y_i - \bar{y})^3}{\{\frac{1}{T}\sum_{i=1}^{T}(y_i - \bar{y})^2\}^{\frac{3}{2}}} \quad \text{And}$$

$$\hat{b}_2 = \hat{\mu}_4 / \hat{\alpha}_4 = \frac{\frac{1}{T}\sum_{i=1}^{T}(y_i - \bar{y})^4}{\{\frac{1}{T}\sum_{i=1}^{T}(y_t - \bar{y})^2\}^2}$$

The JB test statistic is defined as:

$JB = T[\frac{\hat{b}_1}{6} + \frac{\hat{b}_2}{24}]$ Where (T), is the sample size [8].

1) 2.2.3. The Conditional Variance Models

In these models, the key concept is the conditional variance, that is, the variance conditional on the past. In GARCH family models, the conditional variance is expressed as a linear function of the squared past values of the series. Thus, these families of models capture the main stylized facts characterizing financial time series [5]

2.2.3.1. The ARCH Model

The autoregressive conditional heteroscedasticity model for the variance of the errors, denoted by ARCH (q), the conditional variance is given as:

$$\varepsilon_t = \sigma_t v_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon^2_{t-1} + \varepsilon^2_{t-2} + ... + \alpha_q \varepsilon^2_{t-q}$$

This can be also written as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon^2_{t-i}$$

$\sigma^2_t = \text{var}(\varepsilon_t \mid \varepsilon_{t-1}, \varepsilon_{t-2},...)$. This indicates the current value of the variance of the errors possibly depend upon previous squared error terms and the non-negativity constraints can be imposed as, $\alpha_0, \alpha_i > 0 \;\; i = 1,2,...$ [10].

2.2.3.2. The GARCH Model

Given the historical information on the monthly price inflation series as $y_1, y_2,..., y_t$ under the presence of ARCH effect, the GARCH (p q) model can be written as:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon^2_{t-1} + ... + \alpha_q \varepsilon^2_{t-q} + \beta_1 \sigma^2_{t-1} + \beta_2 \sigma^2_{t-2} + ... +$$

GARCH model allows the conditional variance to depend upon previous own lags.

This can also be written as:

$$\sigma^2_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon^2_{t-i} + \sum_{j=1}^{q} \beta_j \sigma^2_{t-j}$$

Restrictions: $\alpha_0 > 0, \alpha_i \geq 0, \beta_j \geq 0$ for $i = 1,2,..., q$ and, $j = 1,2,..., p$. moreover, the stationarity condition for GARCH model is: $\alpha + \beta < 1$ [11].

2.2.3.3. The EGARCH (Exponential GARCH) model

To capture the leverage effect on volatility of the GARCH model in handling financial time series, Nelson proposed the exponential GARCH (EGARCH) model. In particular, it allows for asymmetric effects. EGARCH (p, q) models with mean equation and the variance of residuals at a time t given as:

$$\ln(\sigma^2_t) = \alpha_0 + \sum_{i=1}^{q} \alpha_i \left|\frac{\varepsilon_{t-i}}{\sigma^2_{t-i}}\right| + \sum_{i=1}^{k} \lambda_i \left(\frac{\varepsilon_{t-i}}{\sigma^2_{t-i}}\right) + \sum_{j=1}^{p} \beta_j \ln(\sigma^2_{t-j})$$

The presence of leverage effect can be tested by the hypothesis that $\lambda_i < 0$ and the impact is asymmetric if $\lambda_i \neq 0$ [12].

2.2.4. Procedures of the model application

2.2.4.1. Stationarity of the series

A time series $\{y_t\}$ is said to be stationary if the following three conditions are satisfied.

- $E(y_t) = \mu$
- $E(y_t - \mu)^2 = E(y_{t-s} - \mu)^2 = \sigma_y^2$
- $E(y_t - \mu)(y_{t-s} - \mu) = E(y_{t-j} - \mu)(y_{t-j-s} - \mu) = \gamma(s)$

Let y_t be the price inflation series, then its log-return is given as:

$$r_t = \log y_t - \log y_{t-1} = \log(\frac{y_t}{y_{t-1}}) = \log(1 + \frac{y_t - y_{t-1}}{y_t})$$

The return series displays many of the typical characteristics in financial time series such as volatility, clustering and leptokurtosis [16]

2.2.4.2. Testing for ARCH effects

To apply GARCH family models in any time series, the presence of ARCH effect must be significant or the null hypothesis of no ARCH effect in the residuals of the conditional mean models must be rejected. If the null hypothesis of no ARCH effect is accepted, applying GARCH family models is inappropriate and unnecessary.

2.2.4.3. Lagrange multiplier (LM) test for ARCH effect

This test is used to test significance of serial correlation in the squared residuals for the first (q) lags. The null hypothesis of the test is that there is no serial correlation in the residuals up to a specified order (q). This is given as:

$$H_0 : \alpha_1 = ... = \alpha_q = 0$$

$$H_1 : \alpha_q \neq 0$$

The test statistic is LM = T * R^2 where, $R^2$is obtained from the regression of $\{\hat{\varepsilon}^2_t\}$ on $\hat{\varepsilon}_{t-i}, i = 1,2 ..., q$, where, T is the number of observations.

B. 2.2.4.4. The Model Order Selection Criteria

Determining a proper model lag order for a given time series data needs necessarily applying the ACF, PACF and information criteria's. These statistical measures indicate how the observations in a time series are related to each other. After selecting appropriate lag order, the final models can be selected using information criteria's or penalty function statistics such as Akaike Information Criterion (AIC) or Bayesian Information Criterion (BIC). The AIC and BIC are a measure of the goodness of fit of an estimated statistical model.

$$AIC = -2\ln(\log likekihood) + 2r$$

$$BIC = -2\ln(\log Likelihood) + (r + r(\ln N))$$

2.2.2. Forecasting Evaluation

2.2.2.1. Error Measures

For the comparison of different candidate models, information criteria's can be used for non-nested models with the same distributional assumptions. However, the out of sample estimating performance (forecasting performance) of time series models can be done using forecasting error measure statistics [14].The predicting performance of the model in this study is determined by the use of most popular criterion that are mean absolute error (MAE), root mean squared error (RMSE) and Theil, U coefficient. These error measures statistics are given as follows.

Mean absolute error can be calculated as:

$$MAE = \frac{1}{T}\sum_{t=1}^{T} |\hat{\sigma}_t^2 - \sigma_t^2|$$

Root mean square error can be calculated as:

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\hat{\sigma}_t^2 - \sigma_t^2)}$$

Theil's, (U) statistic can be used as a measure of forecasting accuracy. Like MAPE statistic, high values suggest poor performance in the forecast. Theil's, (U) can be estimated as:

$$U = \frac{\sqrt{\frac{1}{T}\sum_{t=1}^{T}(e_t(l))^2}}{\sqrt{\frac{1}{T}\sum_{t=1}^{T}\sigma_t^2(l)} + \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\sigma^2_{t+1})}^2}$$

If $u = 0$, $\sigma_t^2(l) = \sigma^2_{t+l}$ for all forecasts and there is a perfect fit. On the other hand, if $u = 1$ the performance is not good.

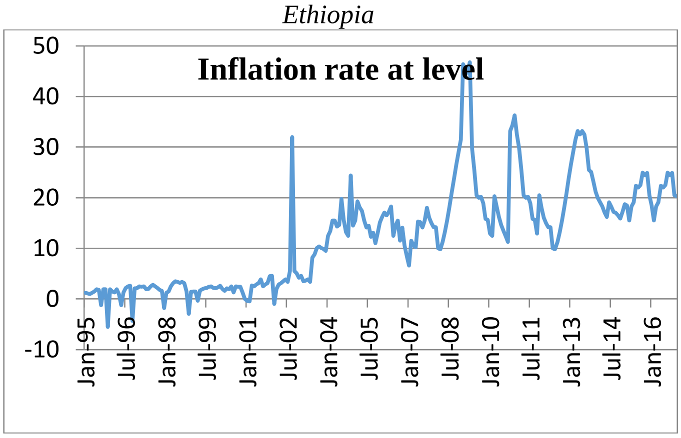
3. RESULTS AND DISCUSSIONS

3.1. Descriptive Analysis

3.1.1. Time series plot of price inflation at level in Ethiopia

The plot of the inflation series at level given below shows that, inflation in Ethiopia rose from around 2002 and there was high fluctuation up to the end of the study period. From January 1995 to April 2003, the series have roughly constant rate and in 2008 and 2011, it has the highest picks of the study period. According to findings of [25], imported food price shocks and poor harvests explain some of the inflation dynamics in both 2008 and 2011. Moreover, the fluctuation for the series has no constant point (mean value). Clearly, it indicates the series is non-stationary at level in both mean and variance, which is tested and presented in Table 2.

Figure 1: *Time series plot of price inflation at level in Ethiopia*



Source: *CSA (Central Statistical Agency of Ethiopia)*

3.2. Tests of Stationarity for Variables of the Study

3.2.1. Unit root test results of price inflation at level

In this study, the stationarity of the explanatory (macro-economic variables) and price inflation series at level and after log-transformation is tested using Augmented Dickey-Fuller and Philips-Paron test statistics. The results of both statistics are presented in Table 2 and 3 as follows.

Table 1: ***ADF and PP-test results of price inflation series at level and log-return***

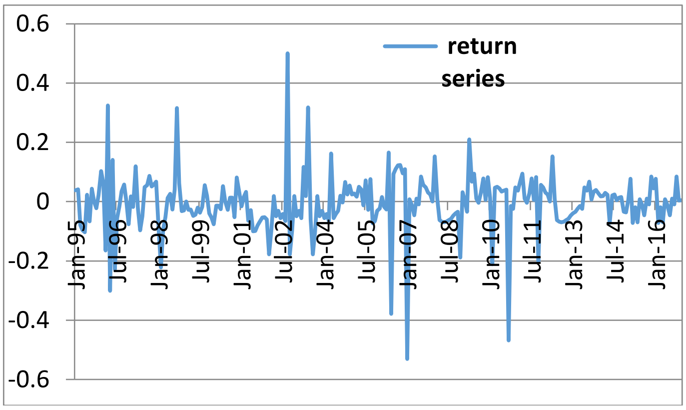
Variable	5%Critical value of ADF test	5% Critical value of PP test	P- value of ADF test	P-value of PP test
Inflation at level	-2.880	-14.00	0.56444	0.3452
Log-return of Inflation	-2.880	-14.00	0.0000	0.0000

As the results of both statistics indicate, there is non-stationarity in price inflation series at level. Moreover, the correlogram of the price inflation at level in Appendix Table 2shows a sample ACF which decays very slowly (the individual ACF are not large) and the sample PACF cuts off after one lag. This indicates transformation or differencing is needed before modelling. The spikes of the moving average cuts off after one because MA models are always weakly stationary since they are finite linear combinations of a white noise sequence for which the first two moments are time-invariant or stationary. There are many mechanisms to change non-stationary series to stationary series. Among those, log-return transformation is applied on price inflation series in this study. Log-return series are easier than price series to handle because return series have more attractive statistical properties such as volatility clustering and leptokurtosis [17]. The series after log transformation is tested again, and the result is presented in Table 3.

3.2.2. Stationarity test of log-return series of price inflation

The results of the both statistics (Augmented Dickey-Fuller and Philips Paron) confirm that the log-return series of price inflation is stationary, because the null hypothesis of non-stationarity is rejected for both statistics. Thus, conditional mean model can be applied in log-return series of price inflation in Ethiopia since it satisfies the basic assumption of the time series or stationarity condition. From the time plot of the return series given below, we can see that there is high fluctuation in Ethiopian price inflation rate. After log-transformation, the series satisfied weak stationarity condition since the time plot of the data is fluctuating with constant variation around affixed level or the series is fluctuating around constant mean value. Moreover, Periods of high volatility is tending to be followed by periods of high volatility and periods of low volatility is tending to be followed by low volatility periods for a prolonged time period. Thus, there is volatility clustering on return series of inflation and the plot indicates high shocks and fluctuations on price inflation in Ethiopia for the study period.

Figure 2: *Time plot of price inflation series in Ethiopia after transformation*



Source: *CSA (Central Statistical Agency of Ethiopia)*

The results of both test statistics in Table 3, indicates that all macro-economic factors or explanatory variables in this study are non-stationarity at level because the null hypothesis of non-stationarity for both Dicky-fuller and Phillips-Peron is accepted. Thus, differencing is required before modeling them [17].

Table 3: *Unit-root tests at level and After first order differentiation for explanatory variables*

Variables at level	5%Critical value of ADF test	5%Critical value	P-value of ADF test	P-value of PP test	
Money supply	-2.879	-2.879	0.8652	0.456	
Exchange rate	-2.880	-2.880	0.9836	0.997	
Lending Interest rate	-2.879	-2.879	0.2048	0.204	
Deposit interest rate	-2.879	-2.879	0.1492	0.148	
GDP	-2.879	-2.879	0.7968	0.694	
After first order differentiation					
Money supply	-2.874	-2.874	0.0000	0.000	
Exchange rate	-2.880	-2.880	0.0000	0.000	
Lending Interest rate	-2.879	-2.879	0.0000	0.000	
Deposit interest rate	-2.879	-2.879	0.0000	0.000	
GDP	-2.879	-2.879	0.0000	0.000	

3.2.3. Stationarity test for explanatory variables after first difference

Macro-economic variables those are tested and presented in Table 4 as non-stationary series are differenced and their Phillips-Perron and Augmented Dickey-Fuller test results are presented in Table 5. The result of both test statistics indicates all of them are stationary after first order difference. This clearly shows all of the macro-economic variables included in this study are integrated of order one. However, strong stationarity is very difficult empirically; Time series must satisfy weak stationarity condition before modeling it [18]. In this study, the stationarity condition is satisfied after first difference for all explanatory variables. Thus, they can be examined as factors for inflation volatility.

3.3. Conditional Mean Model

3.3.1. Model order identification

In this study, the correlogram of inflation after log-transformation is presented in Appendix Table 1.In the correlogram, PACF cuts off after lag three and this indicates the maximum AR lag order must be three. Moreover, the spikes of ACF cut off after two. This suggests the maximum lag order for moving average process must be only two. The well-known Akaike Information Criterion (AIC) and Bayesian Information Criteria (BIC) are applied in this study to select appropriate lag order. In addition to this, plots of ACF and PACF of the log-return series are used as presented in Appendix Table1. From the plot, clearly MA (2) and AR (3) order spikes and probability value of Q-statistics up to lag twenty are significant. Having this, the orders of MA ≤ 2 and AR ≤ 3 are compared using their values of information criteria's i.e. AIC and BIC as presented in Table 6.

Table 2: *Results of Akaike and Bayesian information criteria's for lag orders*

ARMA (p, q)	AIC	BIC
ARMA (0,1)	-3.223	-3.196
ARMA (0,2)	-3.212	-3.185
ARMA (1,0)	-3.241	-3.212
ARMA (1,1)	-3.297	-3.256
ARMA (1,2)	-3.232	-3.191
ARMA (2,0)	-3.238	-3.212
ARMA (2,1)	-3.238	-3.196
ARMA (2,2)	-3.221	-3.194
ARMA (3,0)	-3.267	-3.239
ARMA (3,1)	-3.264	-3.223
ARMA (3,2)	-3.259	-3.218

Among conditional mean models of different lag orders for AR and MA, based on AIC and BIC values of the models, an appropriate lag orders are AR(1) and MA(1) having minimum values for AIC and BIC.

3.3.2. Parameter Estimation of ARMA (1,1) model

After selecting the best conditional mean model order, it is estimated by conditional maximum likelihood parameter estimation method and the result of the estimation is presented in Table 7.

Table 3: *Summary of the results of fitted ARMA (1,1) model*

Model lag variables	Coefficients	p-value
Constant	0.017	0.0000
MA (1)	-0.829	0.0000
AR(1)	0.828	0.0000

In the estimated model result, all lags (lag of moving average and autoregressive) are significant. Thus, the fitted conditional mean model equation can be written as:

$$\hat{y}_t = 0.017 + 0.828 \; y_{t-1} - 0.829 \; \varepsilon_{t-1}$$

3.3.3. Diagnostic Test of the ARMA (1,1) model

3.3.3.1. The Ljung-Box (Q-statistic) Test for Serial Correlation

After estimating conditional mean model, a fitted model must be examined carefully to check for possible model inadequacy. If the model is adequate, then the residual series should behave as a white noise. The plots of ACF, PACF and the Ljung–Box statistics of the residuals can be used to check the closeness of (ε_t) to a white noise(Robert A., 2010).In this study, Ljung-Box (Q-statistic), plots of ACF and PACF are applied to test the residuals from ARMA (1,1) and the result is displayed in Table 8 as follows.

Table 4: *Results of Q-statistics for conditional mean model residuals*

Lag order	Q-statistics	P- Value
1	0.15	0.69
2	0.47	0.79
3	0.52	0.91
4	1.74	0.78
5	1.75	0.88
6	7.36	0.28
7	7.62	0.36
8	10.93	0.21
9	10.94	0.28
10	10.99	0.3
11	13.70	0.24
12	13.78	0.31
13	14.10	0.36
14	14.44	0.41

The result of the Ljung-Box statistics given above does not reject the null hypothesis of no serial correlation in the residuals of the conditional mean model because its probability value is greater than 0.05 up to lag order 14. Moreover, the spikes of the ACF and PACF in Appendix Table 1are clearly insignificant and this suggests the residuals of the conditional mean model are white noise. Thus, ARMA (1,1) is the best conditional mean model for the return series of the price inflation in Ethiopia.

3.3.3.2. Normality test

The well-known Jarque-Bera normality test statistics is applied in this study to get the results of skewness, kurtosis and the p-value of the test statistics as presented in Appendix Figure 1.The Jarque-Bera test statistics result, which is presented in Appendix Figure 1, clearly shows, the null hypothesis of normality of the residuals is accepted. Therefore, the residuals of the ARMA (1,1) model is serially uncorrelated and normally distributed random variables.

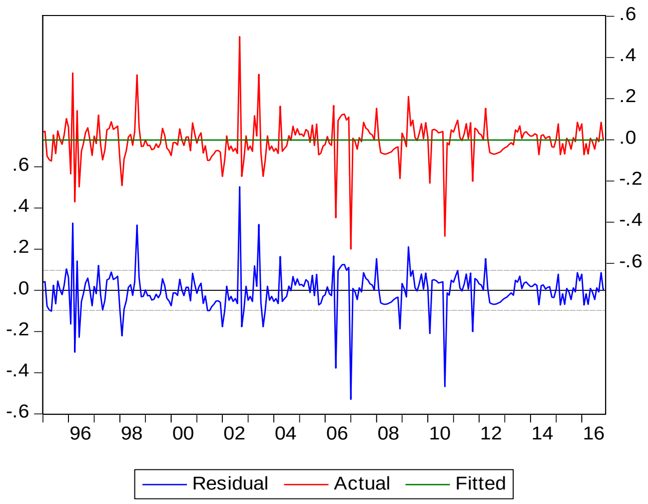
3.4.2.3. Test for ARCH effects (ARCH- LM test)

Table 5: *The result of Lagrangian Multiplier test for the existence of ARCH effect*

F-statistic (11.1203)	**p-value** 0.000		
Lag order	Coefficient	t-statistic	p-value
1	0.125	2.130	0.034
2	0.047	0.784	0.433
3	0.144	2.570	0.010
4	0.049	0.875	0.382
5	0.241	4.286	0.000

After correctly fitting the conditional mean model, the residuals are tested for the existence of the ARCH effect. Lagrangian multiplier test statistics is applied in this study to test the conditional hetroscedasticity and the test result confirms that there is significant ARCH effect in the residual of the conditional mean model of price inflation return series in Ethiopia because the null hypothesis of no ARCH effect in the series is rejected. However, the Q-statistics indicates that the residuals of the ARMA (1,1) model no longer shows any significant autocorrelation. On the other hand, Figure 3shows highly significant autocorrelation between the squares of these residuals. This indicates dependency in the second moments of the residuals, which contradicts the assumption of a constant, time-invariant variance. Thus, it clearly shows the existence of ARCH effect in the return series in addition to LM test statistics. Therefore, the price inflation in Ethiopia is volatile and it is appropriate and relevant to apply conditional variance or GARCH family models.

Figure 3: *The time plot of the squared residuals from ARMA (1,1) model*



Since the ARCH effect is significant, to correctly specify conditional variance model the ACF and PACF plots of the squared residuals from the conditional mean model is presented in Appendix Table 4. In the correlogram, there are significant spikes suggesting serial correlations on the series. In addition to plots of ACF and PACF, the Lijung-Box or Q-statistics indicates there is significant serial correlation in the squared residuals of the conditional mean model up to lag 15 as follows.

Table 6: *Summary result of Q-statistic for squared residuals from ARMA (1,1) model*

Lag order	Q-statistics	P- Value
1	8.47	0.00
2	14.93	0.00
3	20.56	0.00
4	24.61	0.00
5	42.97	0.00
6	54.69	0.00
7	57.21	0.00
8	66.31	0.00
9	74.33	0.00
10	83.83	0.00
11	91.86	0.00
12	91.83	0.00
13	92.46	0.00
14	93.06	0.00
15	104.46	0.00

However, specifying the order of a GARCH model is not easy. Only lower order GARCH models are used in most applications, such as GARCH (1,1), GARCH (2,1), and GARCH (1,2) models. For the comparison of the different ARCH (q) and GARCH (p) orders, the combination of lag orders ≤ 2 are compared in this study because the effect of ARCH and GARCH is less for higher lag orders [7]. To select appropriate ARCH and GARCH lag orders, information criteria's (AIC and BIC) are applied in this study. The lag order having minimum values for information criteria's is selected as appropriate order and the comparison is presented in Table 11.

3.4. Application of Conditional Variance Model

3.4.1. Lag order selection for conditional variance model

The Q-test statistics of the squared residuals of the mean model in Appendix Table 4 suggests that all spikes are significant but it is better to consider lag order ≤ 3 because higher lag orders may have lower effects than lower lag orders [21]. From the given values of AIC and BIC we can see that GARCH (1,1) with normal distribution, GARCH (1,1) with GED distribution, GARCH (1,1) with t-distribution, EGARCH (1,1) with GED distribution, EGARCH (1,1) with normal distribution and EGARCH (1,1) with t-distributional assumption are selected as appropriate models having small information criteria value. The next thing we do after selecting candidate models having different distributional assumptions is measuring their forecasting abilities using MSE (mean square error), RMSE (root of mean square error) and U thel's inequality coefficient. The best model among the candidate models used for estimating and forecasting the volatility in return series of price inflation in Ethiopia.

3.4.2. Comparing GARCH family models using forecasting error measures

The results of forecasting error measures in Table 12 clearly shows that ARMA (1,1)-GARCH (1,1) with GED distributional assumption for residual has smallest error measures.

Table 7: *The results of forecasting error measures for GARCH family models*

Models	MAE	RMSE	Thail's (U)
GARCH (1,1) with Normal distribution	0.0371	0.04805	0.7033
GARCH (1,1) with GED distribution	0.0362	0.04802	0.6985
GARCH (1,1) with t-distribution	0.0363	0.04809	0.7067
EGARCH (1,1) with Normal distribution	0.0363	0.04803	0.7103
EGARCH (1,1) with GED distribution	0.0365	0.04896	0.7333
EGARCH (1,1)with t-distribution	0.0363	0.04812	0.7014

Although, GARCH model is parsimonious than ARCH, it assumes equal weight for negative and positive economic shocks. In practice, negative shocks (bad news) have more effect on economic volatility than positive shocks (good news) this is known as leverage effect [21]. In this study, GARCH model is extended to EGARCH (exponential Generalized autoregressive conditionally hetroskedastic) model to test the existence of leverage effects in the return series of price inflation in Ethiopia. However, it is confirmed that there is no leverage effect because the parameter or (λ_i) is not significant in estimated EGARCH model. This indicates the effect of the positive and negative shocks of inflation are equal on price inflation volatility [22]. Thus, ARMA (1,1)-GARCH (1,1) with GED distributional assumption is an appropriate model for forecasting and estimating return series of the price inflation volatility in Ethiopia.

3.4.3. Estimating parameters of ARMA (1,1)-GARCH (1,1) model

After the existence of the significant ARCH effect in the conditional mean model, the volatility and mean model equations are estimated jointly and the result is presented in Table 13.

Table 8: *Summary of the fitted conditional variance or GARCH (1,1) model*

ARMA (1,1)		
Lags	Coefficients	p-value
AR (1)	0.701	0.000
MA (1)	-0.580	0.000
GARCH (1,1) Model		
Lags and variables	Coefficients	p-value
Constant	0.000	0.226
ARCH (1)	0.085	0.041
GARCH (1)	0.811	0.000
Exchange rate	0.080	0.000
Lending interest rate	0.185	0.040
Deposit interest rate	-0.057	0.417
GDP	0.047	0.404
Money supply	0.212	0.000

The finding of this study shows that USD exchange rate has significant and positive contribution to inflation volatility in Ethiopia because the parameter is positive and statistically significant at 1% level. This finding is consistent with finding of [23] which concluded that the level of the exchange rate matters for the economy's inflationary pressures because an appreciating exchange rate is usually thought to be deflationary and depreciating exchange rate is usually thought to be expansionary and inflationary. Lending interest rate is also significant macro-economic factor of inflation volatility in Ethiopia. In line with this [4] concluded that there is a long-run co-integrating relationship between lending interest rate and expected inflation.In addition to this, Money supply has positive relationship with inflation volatility in Ethiopia because the parameter is positive and statistically significant at 1% level. This finding is consistent with the finding of [25] which concluded exchange rate and money supply as potential sources of recent surge in inflation in Ethiopia. In line with this, [3] concluded that the main determinants of inflation in Ethiopia are depreciation in Ethiopian Birr and increase in Broad money supply. Macro-economic variables, which are significantly affecting price inflation volatility are also have direct contribution to price inflation this is because higher inflation levels are typically associated with higher inflation volatility [27]. From the given fitted model output, estimated conditional variance model equation is obtained as:

$$\hat{\sigma}^2_t = 0.000548 + 0.08506\ \varepsilon^2_{t-1} + 0.8115\ \sigma^2_{t-1}$$

Moreover, the result of the variance equation shows that the coefficient of the ARCH and GARCH term are statistically significant. This shows current month price inflation volatility was affected by its last month lagged shocks positively. Similarly, the GARCH term is statistically significant. This indicates current month price inflation volatility was affected by its last month lagged price inflation volatility positively. Moreover, in the estimated variance equation, the existence of unconditional variance of (ε_t) is confirmed because

$$\alpha + \beta = 0.8965$$

This condition satisfies the weak stationarity of GARCH model since $\alpha + \beta$ is less than one. In the GARCH(1,1) model, the larger values of α leads to greater volatility in the forecasted errors, while high values of β indicate higher persistence. Thus, the effect of the current shock and volatility are not persistent in the future volatility. If $\alpha + \beta = 1$, then conditional variance grows linearly with the forecast horizon (Amadeus, 2014).Before forecasting using ARMA (1,1)-GARCH model, the residuals must be tested to check the adequacy of the model (Amadeus, 2014). In this study, the residuals of ARMA (1,1)-GARCH (1,1) model are tested as follows.

3.4.4. Diagnosis test of ARMA (1,1)-GARCH (1,1) model

After fitting conditional variance model or ARMA (1,1)-GARCH (1,1), the residual are tested for existence of the remaining ARCH effects on the residuals. The usual test statistics called LM-test for ARCH effect is used in this study. If there is remaining ARCH effect on the residuals, the model is not adequate and it must be corrected by adding more lags (Amadeus, 2014). In addition to this, the serial correlation of the residuals of the conditional variance model is tested. In this study, Q-statistics (Ljung-Box) is used to test the serial correlation in residuals of the conditional variance or ARMA (1,1)-GARCH (1,1) model. If there is serial autocorrelation in the residuals of the fitted model residuals, the model is not adequate and it must be corrected. The results of the both test statistics are presented in Table 14 and 15.

3.4.4.1. Testing for the remaining ARCH effect

In the result given in Table 14, we can see that the probability value of the ARCH-LM test for ARCH effect confirms there is no remaining ARCH effect in the residual of the fitted conditional variance model because the null hypothesis of the no ARCH effect in the residuals is accepted. If the null hypothesis of no ARCH effect in the residuals was rejected, GARCH (1,1) model is inadequate and it must be re-specified because the residuals are serially correlated. Thus, GARCH (1,1) model with GED distributional assumption best fitted return series of the price inflation in Ethiopia.

Table 9: *Table of the results of remaining ARCH effect in residuals of ARMA (1,1)-GARCH (1,1) model*

F-statistic (1.348)	p-value 0.244		
Lag orders	Coefficient	t-statistic	p-value
1	0.000	0.012	0.990
2	0.034	0.549	0.582
3	-0.028	-0.456	0.648
4	-0.154	-2.499	0.113
5	-0.011	-0.175	0.861

4.4.4.2. Testing for the serial autocorrelation in the residuals

Q-statistics for serial correlation of the conditional variance model equation clearly shows us the null hypothesis of no serial correlation is accepted up to lag fifteen. Thus, GARCH (1,1) with GED distributional assumption well fitted return series of price inflation series in Ethiopia.

Table 10: *The result of Q-statistics of the residuals of conditional variance model*

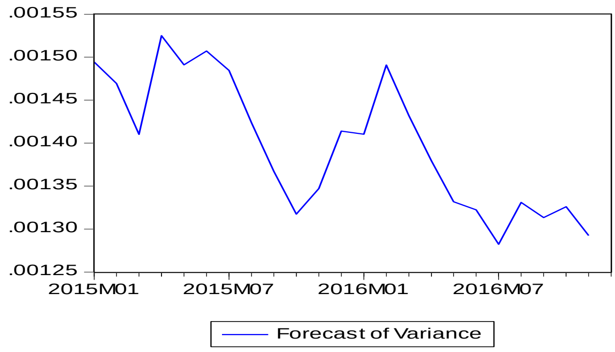
Lag order	Q-statistics	P-Value
1	1.47	0.22
2	3.16	0.21
3	4.32	0.22
4	5.15	0.27
5	6.26	0.28
6	8.68	0.19
7	9.76	0.21
8	14.92	0.06
9	15.54	0.07
10	15.53	0.11
11	17.50	0.09
12	18.32	0.11
13	18.62	0.13
14	18.64	0.18
15	18.64	0.23

The result of the VIF statistics clearly shows that there is no multicollinearity problem in the data available for explanatory variables because all explanatory variables have VIF values which are less than 10.

3.5. Forecasting Price Inflation Volatility in Ethiopia

ARMA (1,1)-GARCH (1,1) with GED distributional assumption has been selected as best model because it satisfied all assumptions of the model adequacy tests. Thus, it is used as the final model for forecasting return series of price inflation in Ethiopia. In this study, the whole available data is splinted in to two categories as estimation and evaluation periods even if no guidance exists on how to choose the split point. The best splitting point or forecasting origin is the one with smallest error value. To perform forecast of the variance, the data from January 1995 up to January 2014 was used for estimation and January 2015 up to January 2016 used for forecasting because in the Table 17 the splitting point January 2015 has minimum values for all forecasting error measure statistics.

Figure 4: *Out of sample forecast graph of price inflation volatility in Ethiopia*



The forecasted graph of the price inflation volatility clearly shows there was high inflation volatility in Ethiopia up to December 2015. From January 2016 to July 2016, there is high decrement and starting from August 2016 up to the end of the study period there is roughly constant inflation volatility. This finding is consistent with the finding of [25] which concluded that due to the attention that Ethiopian government has been paying since 2008 on monetary targeting to reduce high inflation rate in Ethiopia, there is decrement on inflation volatility in Ethiopia. The result of forecasted value is presented in Appendix Table 8.

4. CONCLUSIONS AND RECOMMENDATIONS

4.1. Conclusions

To select the best model among different candidate models with similar distributional assumptions, well-known information criteria's (AIC and BIC) were applied. The final best model for estimation and forecasting price inflation volatility was selected using different forecasting abilities of the models. Usual forecasting error measures such as, root mean square error (RMSE), mean absolute error (MAE) and Uthail's inequality coefficient were applied and finally ARMA (1,1)-GARCH (1,1) model with GED or Generalized Error distributional assumption was selected having smallest values for all error measures and Uthail's inequality coefficient. Different macro-economic factors (exchange rate, lending interest rate, deposit interest rate, GDP and Broad money supply) for inflation volatility in Ethiopia are examined statistically. The result clearly showed except GDP and deposit interest rate, others are significantly contributing factors for the volatility of price inflation in Ethiopia. The finding of this study confirms that there is significant positive relation-ship between broad money supply, which is financed by government and inflation volatility in Ethiopia. Interest rate is examined as lending and deposit interest rate. The finding showed that, lending interest rate has significant relation-ship with price inflation volatility in Ethiopia and deposit interest rate is insignificant. The result also showed that exchange rate has significant relation-ship with price inflation volatility in Ethiopia. Moreover, the shocks and volatility in last time have significant impact on inflation volatility. Finally, forecasted graph clearly showed that, there is high fluctuation of variance (volatility) and it has low rate at the end of the study period because nowadays authorities of Ethiopia sharpened the focus on monetary targeting, adopting high-powered money as nominal anchor to reduce price inflation rate in Ethiopia.

4.2. Recommendations

Based on the findings, the followings are some of the recommendations.

- This study suggests that, to come up with stable price inflation volatility in Ethiopia, the government, policy makers and scholars must pay great effort to control macro-economic factors, which has direct contribution on volatility.
- Government may intervene to control Money Supply and Interest Rates through monetary authority (National Bank).
- Since, high Interest Rate reduces investments and low Interest Rate results in high inflation and volatility rate, monetary authorities must keep the balance between these variables.
- Monetary authority (National Bank) can intervene in balancing Exchange Rates through some mechanisms of controlling.
- To overcome the future volatility and shocks of inflation, current volatility must be controlled since terms of volatility and shocks have positive effect on current volatility.
- Also recommends other researchers and scholars to use Multivariate GARCH or other GARCH family approaches to model financial volatilities including price inflation volatility.

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APPENDIX

Table 11: The results of the AIC and BIC for different lag orders of conditional variance model

GARCH models with different distributions	AIC	BIC	EGARCH models with different distributions
GARCH(1,1) with normal distribution	-3.547*	-3.411*	EGARCH (1,1) with normal distribution
GARCH (1,2) with normal distribution	-3.540	-3.391	EGARCH (1,2) with normal distribution
GARCH (2,1) with normal distribution	-3.529	-3.380	EGARCH (2,1) with normal distribution
GARCH (2,2) with normal distribution	-3.526	-3.363	EGARCH (2,2) with normal distribution
GARCH(1,1) with GED-distribution	-3.529*	-3.380*	EGARCH (1,1) with GED-distribution
GARCH (1,2) with GED-distribution	-3.527	-3.365	EGARCH (1,2) with GED-distribution
GARCH (2,1) with GED-distribution	-3.510	-3.347	EGARCH (2,1) with GED-distribution
GARCH (2,2) with GED-distribution	-3.512	-3.336	EGARCH (2,2) with GED-distribution
GARCH(1,1) with t-distribution	-3.522*	-3.373*	EGARCH (1,1) with t-distribution
GARCH (1,2) with t-distribution	-3.516	-3.354	EGARCH (1,2) with t-distribution
GARCH (2,1) with t-distribution	-3.506	-3.344	EGARCH (2,1) with t-distribution
GARCH (2,2) with t-distribution	-3.502	-3.326	EGARCH (2,2) with t-distribution

Notes: *, indicates the minimum value of AIC (Akeike Information Criteria) and BIC (Bayesian Information criteria) for models with similar distributional assumption.