Enumerating Of Star-Magic Coverings And Critical Sets

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Abstract: A simple graph $G = (V, E)$ admits a H-magic covering if every edge in G belongs to a subgraph of G that isomorphic to H. Let $K_{m,n}$ be a complete bipartite graph on m and n vertices. Then graph G is a $K_{1,n-magic}$ if there is a total labeling $f: V \cup E \to \{1, 2, \dots |V| + |E|\}$ such that for each subgraph $G' = (V', E')$ of G isomorphic to $K_{1,n}$, there exists $\sum_{v \in V'} f(v) + \sum_{e \in E'} f(E) = m(f)$, where m(f) is a positif integer called magic sum. When f(V)= $\{1, 2, \dots |V|\}$ we say that G is $K_{1,n-super\ magic}$ and we donete its supermagic-sum by s(f). The aim of this research is to list all possible H-magic covering on complete bipartite graph, where H is star magic, especially for n=3. After we list all possible star magic covering on complete biparite graph, we tke s one example of the labeling to find the critical sets. The method f this research is a literary study and we carried outthe enumertion of all possible existing labeling. This research get a sum of 836 labeling which are not isomorphic. After we get list of all possible labeling, we choose a set of label vertices {1,3,4,5,7,10} from which we establish some 28 critical set, 21 of them are of size 1 and the others are of size 2

Keywords: $K_{1,n} - magic\ covering$, critical sets, complete bipartite graph

1. INTRODUCTION

Graph labeling is an interesting topic in graph teory. Since Rosa [8] introduced the graceful labeling, various types of labelling studied and developed (see[3]). According to Wallis [11], the most common choices of the domain are the set of all vertices and edges (such labelings), the vertex-set alone (vertex-labelings), or the edge-set alone (edge-labelings). One type of graph labeling that much studied and developed is a magic labeling. The idea of labeling was first introduced by Sedlacek [10], then formulated by Kotzig and Rosa [9]. One of the magic labeling that frequently discussed is the edge-magic total labeling. From the development of edge-magic total labeling, Gutierrez dan Llado [4] has faound a magic covering. Most of the notations and terminologies in this paper follow that of Bondy and Murty [2]. Let $G = (V, E)$ be a finite simple graph. Gutierrez and Llado [4] defined edge-covering of G as a family of different subgraph $H_1, \dots, H_k$ such that any edge of E belongs to at least one subgraphs $H_i, 1 \leq i \leq k$. Then, it is said that G admits an $(H_1, \dots, H_k) - (edge)$ covering. If every H_i is isomorphic to a given graph H, then we say that G admits an H-covering. Suppose that $G = (V, E)$ admits an H-covering. We say that a bijective function $f: V(G) \cup E(G) \to \{1, 2, \dots, |V(G)| + |E(G)|\}$, is an H-magic labeling of G if there is a positive integer m(f), which we call magic-sum, such that for each subgraph $H' = (V', E')$ of G isomorphic to H we have,

$$f(H') = \sum_{v \in V'} f(v) + \sum_{v \in E'} f(e) = m(f)$$

In this case we say that graph G is H-magic (Gutierrez and Llado [4]). Gutierrez and Llado [4] proved that a complete bipartite graph $K_{n,n}$ can be covered by $K_{1,n} - star\ magic$ covering using a magic square to label vertices and edges. Further magic covering had been investigated and developed by Llado and Moragas [5], T.K. Maryati et al [6], and .A.A.G. Ngurah et al. [7]. This research is the expanding of the results of Guiterrez and Liado [4] to list all possible star magic coverings on complete bipartite graphs. The methodology applied considered different from [4]. After gained all possible labeling, we worked on finding some critical sets of a chosen labeling. Critical sets on a graph is a subset of labels and positions which when completed will produce in a single labeling (Baskoro[1]). Critical sets can be applied to security in communication systems.

2. BIPARTITE GRAPH $K_{2,2}$

Sum Label Verte x (T)	Sum Ajaib [illegible]	Label Vertex				Label Edge			
		A	b	C	d	e	f	g	h
12	21	1	5	4	2	8	6	3	7
15	22	7	1	2	6	4	3	8	5
20	23	2	8	7	3	5	6	1	4
24	24	8	4	5	7	1	3	6	2

To bipartite graph $K_{2,2}$ as follows 4 are:

To ease writing, it make set of ctitical sized -1 and can write as : Q11= {(7,7)}, Q12 = {(4,6)}, Q13 = {(4,3)} and Q14 = {(8,2)}.

Then, there are 54 set of critical sized 2 with 4 of the first are Q21={_, 2,_,_,_,1,_}, Q22={_,_,_,_,_,1,3,_}, Q23={1, _,4,_,_,_,_,_}, Q34={_, _,_,_,_,4,1,_}, that can wrote as ease as Q21={(2,2), (7,1)}, Q22={(6,1), (7,3)}, Q23={(1,1), (3,4)}, and Q24={(6,4), (7,1)},

If taken example set of critical Q24 as example, so:

S = {(1,1), (4,2), (6,8)}
Or S = {1, 0, 0, 2, 0, 8, 0, 0}

So third participant in enumerating star magic to access savings account is with maked third any code which if it summed mod 9 can produce S as above

P1 = {2, 3, 7, 5, 1, 1, 4, 0}
P2 = {4, 0, 2, 2, 3, 6, 1, 7}
P3= {4, 6, 0, 4, 5, 1, 4, 2}

3. COMPLETE BIPARTITE GRAPH $K_{3,3}$

For complete bipartite graph $K_{3,3}$ can be obtained enumerating star magic $K_{1,3}$ as follows:

T	21	24	27	30	33	36	39	42	45	48
S(f)	47	48	49	50	51	32	53	54	55	56
Any labels	0	0	0	31	0	109	0	51	0	77

And

T	51	54	57	60	63	66	69	72	75
S(f)	57	58	59	60	61	52	63	64	65
Any labels	0	51	0	109	0	31	0	0	0

Some results of enumerating star magic of complete bipartite graph for T=30, s(f)=50

4	5	6	12	1	2	10	14	7	8	9	13	15	11	3
4	5	6	12	1	2	10	14	7	8	9	13	11	15	3
4	5	6	12	1	2	8	10	13	14	9	7	11	15	3

4. COMPLETE BIPARTITE GRAPH $K_{4,4}$

For complete bipartite graph $K_{4,4}$ can be obtained enumerating star magic $K_{1,4}$ as follows:

T	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
S(f)	90	93	96	99	102	105	108	111	114	117	120	123	126	129	132	135
Any labels	On process															

Some of enumerating star magic on complete bipartite graph for T=48, s(f)=93

11	9	1	3	4	5	7	8	20	17	19	2	15	13	10	22	6	18	21	23	24	16	12	14
4	5	8	7	9	1	11	3	20	10	16	19	24	13	15	12	14	23	6	18	2	22	21	17

5. ILLUSTRATIONS

In this case , we search a – star magic covering on bipartite graph with T-30, s(f)-50, and s-90, and costruct its dual labeling. For T=30 we obtain 23 combinations of the set of label of vertices, to of which are {1,2,3,4,15} and {1,2,3,4,8,12}. Here are the steps to get a star magic covering.

[1] First, we determine the label of each vertex of the combination of label T , for example the set of the label of the vertices obtained is {1,2,3,4,8,12}
[2] Second, we label each edge by labels not use by T, for example the label of edge are 5,6,7,9,10,11,13,14, and 15
[3] Third, we interchange each label of edge, until we get a star magic covering . in the process of interchange the labels of edges we can obtain more than one labelling.
[4] Next, we will search the dual labeling. For examples, the labeling with T=30, s(f)=50, s=90 has a dual labeling with T=66, s(f)=62, s=54.

6. CONCLUSION

The result obtained from the research that have been done are as follows.

[1] There are 836 non isomorphic –magic covering of T-21 to T-75
[2] From a chosen set of label of vertices {1,3,4,5,7,,10} there are 28 critical sets, 21 of them are of size 1 and the other are of size 2

Star magic covering on complete bipartite graph with T=30, s(f)=50, and s=90 and its dual labeling with T=66, s(f)=62 and s=54

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