Optimization of Imperfect Manufacturing Systems: Interference Analysis Between Stochastic Flow of Failures and Production Policy

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Abstract: This article presents a problem of optimizing the optimal stochastic control policy of a manufacturing system operating in an uncertain environment. The manufacturing unit or machine can only produce one type of part at a time. The objective of the study is to develop an optimal joint production and maintenance planning strategy aimed at making the investment profitable (minimizing the total cost incurred) while satisfying a given demand. The manufacturing unit is subject to random breakdowns and repairs. The start-up time is negligible compared to the manufacturing time of a type of part and the average time between the arrival of requests (production flexibility). In addition, the cost of running is also negligible compared to the costs of inventorying, shortage and repair of the machine. A modeling approach based on stochastic control theory and an algorithm for numerical resolution of optimum conditions are presented. Finally, the study's contribution is to find optimal production and maintenance policies that are more improved than the Modified Hedging Corridor Policy (MHCP).

Keywords: optimal control, numerical methods, production systems, production rate, maintenance policy.

1. Introduction

This article discusses the class of imperfect manufacturing systems that involve non-flexible machines and aims to analyze the interactions between production control and Setup policies. It builds on the structure of production and setup policy already developed in the scientific literature by Bai and Elhafsi [1], Boukas and Kenné [2], and Hajji et al. [3],

In the field of research into optimal production control policies for the different classes of manufacturing systems subject to random breakdowns and repairs, and in the case of production planning of a manufacturing system; composed of a single machine capable of producing a single type of product and whose dynamics are described by a homogeneous Markov chain (constant transition rates), the work of Kimemia and Gershwin [4] and Akella and Kumar [5] has demonstrated that the Hedging point policy (HPP) is optimal. Sethi and Zhang [6] showed a detailed formulation of the problem of optimal control of a production system consisting of a machine and producing several types of products, neglecting the setup. Boukas and Haurie [7] proposed to use the numerical method described in Kushner and Dupuis [8] Gharbi and Kenné [9] posed the same hypothesis to study the problem of production control for a manufacturing system with several machines and several products (MnPn). Bai and Elhafsi [1] conceptualized the optimal conditions for a non-flexible manufacturing system consisting of a machine and producing two types of products, cost and time to run (setup) not negligible. They found the heuristic solutions to the problem Then they presented an adapted structure of the control policy, called Hedging Corridor Policy (HCP). Detale problem analysis using stochastic optimal control methods and formalism of Hamilton-Jacobi-Bellman equations (HJB) and presented in Hajji, Boukas and Kenné [10], Hajji, Gharbi and Kenné [3], extended the results of Bai and Elhafsi [1], and obtained an improved suboptimal control policy called MHCP (Modified Hedging Corridor Policy).

In the industrial context, Setup operations generally entail significant time and costs. The latter can have a significant impact on the competitiveness of the manufacturing company, whose flexibility and performance of its production system are limited by costs and setup shutdowns without added value. Indeed, Setup's actions generate significant losses that must be minimized, such as those of the operating cost of the machines, the time recorded by human resources and the essential time between the beginning of a setup operation and the beginning of production. Hence the need to control and reduce the number of Setup actions. It is a question of setting up a global and efficient structure of setup operations by considering all the interactions between the partial costs of the system such as the cost of production, inventory, out of stock, etc. Thus, the goal is to use an optimal production and setup control policy that increases productivity and resource availability as well as minimizes the total cost incurred from the system.

The main objective of this article is a continuation of the work carried out by Bai and Elhafsi [1], Boukas and Kenné [2], kibouka et al. [11], Our job is to compare different assumptions that can describe the production and maintenance procedure. It is demonstrated how hypotheses affect optimality conditions that then transform into HJB equations (specific for each case) and in numerical solution algorithms accordingly. We then compare the optimal production and maintenance policies obtained in accordance with maintenance rules.

This paper is organized as follows: Section 2 presents the notations and main assumptions of the proposed model. Section 3 presents the statement of the optimal production and setup scheduling problem. The optimality conditions and numerical approach are presented in Section 4. Section 5

describes the numerical example with results analysis, and the paper is concluded in Section 6.

2. Model Assumptions and Hypotheses

This section presents the table of notations and assumptions used throughout this paper.

2.1 Notation

Table 1: Notation

Notation	Designation
$u_i(t)$	Production rate of part type i
$\overline{u_i}$	Max production rate of part type i
u_r	Maintenance policy
τ_i	Production time of product i
d_i	Rate of P_i product request
$\alpha(t)$	Stochastic process describing the dynamic of the machine M_1P_n
$x_i(t)$	vector inventory levels/shortage, product type i
B	Space of possible modes of the system $= \{0,1\}$
$q_{ij}(.)$	transition rate, mode $\forall\ (i,j) \in B$
$MTTF$	mean time to failure
$MTTR_{max}$	Mean Time To Repair max
$MTTR_{min}$	Mean Time To Repair mIN
c^+	Inventory cost, product type i,
c^-	Shortage cost, product type i
c_r	Coût de réparation de la machine
ρ	Cost discount rate

2.2 Context and Assumptions

The following is a summary of the general context and main assumptions considered in this paper:

- As we have observed in the topic itself, the instant cost $g(.)$ will consist of the cost of putting in inventory, the cost of shortage and the cost of repairing the machine, related to the repair time. We therefore neglect the production costs of each product, but also the costs and setup times when changing the part to be produced on the machine. This assumption is made in the statement and we accept it since it is likely that this cost will be lower than the costs previously considered in the definition of instantaneous cost
- We consider that the input raw material of our open chain is infinite. This hypothesis, in an open loop as here, is not a hypothesis usually too strong to describe a system.
- We posit that the horizon of the study is infinite, for reasons of simplification of the construction of the model, and also because on the scale of the production time, the production strategy that we develop on a much larger horizon, a few years.
- We also hypothesize that thanks to observations in production we know the maximum production rates that the machine can achieve for each type of part.
- We consider the failure rate as constant (invariant with age) as well as the demand. The case of a variable request will be discussed a posteriori of the study that will follow.
- Finally, to manage corrective maintenance, we posit that the repair rate is variable and can take two values: either max or min. In this way we will be able to obtain the maintenance policy in addition to the production policies.

3. Problem Statement

The production system considered represents a common problem in the manufacturing industry. The system consists of a single non-flexible machine producing only one type of product at a time. This machine is subject to random breakdowns and repairs that can lead to backlog situations. These decision variables respectively influence the inventory and system capacity. The structure of the system studied is shown in Fig. 1. The stochastic process resulting from this integration is then a transition rate-controlled process (non-homogeneous Markov process.

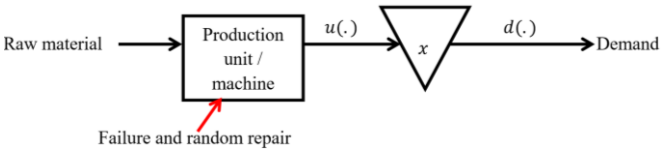

Figure 1: Manufacturing system studied

Knowing that $u \in \mathbb{R}^n$ is the vector of production policies for each type of product, $d \in \mathbb{R}^n$ is the vector of applications for each type of product and $x \in \mathbb{N}^n$ is the vector of product stocks. To be able to model this production system mathematically, it takes assumptions, which we will now state.

We assume that these two variables are distributed exponentially with ratio ***p*** et ***r***, respectively.
These hypotheses make it possible to describe the dynamics of the state of the machine α (t) by a jumping process $\{\xi(t), t \geq 0\}$ corresponding to the discrete state of the machine. This discrete state is generated by a continuous-time, discrete-state Markovian process, called mode, taking its values in $M = \{1,0\}$, defined as:

$$\xi(t) = \begin{cases} 1 & \text{if the machine is in the operaional mode} \\ 0 & \text{if the machine is in the failure mode} \end{cases} \quad (1)$$

Transition diagram of a Markov string (two states)

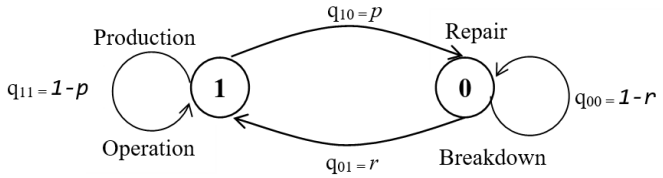

Figure 2: State transition diagram of the stochastic process

Here the transition ratio $q_{01}(.)$ is a decision variable that can take the following two values:

$$q_{01}(.) = u_r = \begin{cases} u_{r,min} = \underline{u_r} = {}^{1}/_{MTTR_{max}} \\ u_{r,max} = \overline{u_r} = {}^{1}/_{MTTR_{min}} \end{cases} \quad (2)$$

As for the failure rate, considered constant: $q_{10} = {}^{1}/_{MTTF}$ in a classic way.

1 Transition ratio matrix

$$Q = \begin{bmatrix} q_{00} & q_{01}(.) \\ q_{10} & q_{11} \end{bmatrix} \text{ avec } q_{ii} = -\sum_{\substack{j=1 \\ i \neq j}}^{2} q_{ij} \quad (3)$$

Knowing that by definition:

$$q_{ij}\delta t + o(\delta t) = P[\alpha(t+\delta t) = j \,|\alpha(t) = i] \quad \forall (i,j) \in B \quad (4)$$

We choose not to replace q_{ij} in this matrix to remain in the general case when writing the equations of HJB.

2 Area of Eligibility

In this case the area of eligible orders includes the production policies for each type of parts but also the corrective maintenance policy of the system, which gives us:

$$\Gamma(\alpha) = \left\{ \begin{array}{c} (u, u_r) = \\ (u_1, u_2, \dots, u_n, u_r) \in \mathbb{R}^{n+1} / 0 \leq u_i \leq \overline{u_\iota} \\ \underline{u_r} \leq u_r \leq \overline{u_r} \\ \forall\, i \in [1;n] \end{array} \right\} \quad (5)$$

3.1 Limit probabilities

By definition of the Chapman-Kolmogorov equations in steady state we can directly write:

$$\begin{cases} \Pi Q = 0 \iff [\Pi_0 \quad \Pi_1] \begin{bmatrix} q_{00} & q_{01}(.) \\ q_{10} & q_{11} \end{bmatrix} = 0 \\ \sum_{i=0}^{1} \Pi_i = 1 \end{cases} \quad (6)$$

It is possible to find the exact expressions with the MATLAB software, in symbolic form, but we will not insert them here because they are not necessary for the rest

3.2 Feasibility conditions

As we are on the horizon, the feasibility conditions only concern the ability of our system to produce, for each type of part, more than the demand, which translates as follows:

$$\begin{cases} \Pi_1 \tau_1 \overline{u_1} > d_1 \\ \Pi_2 \tau_2 \overline{u_2} > d_2 \\ \vdots \\ \Pi_n \tau_n \overline{u_n} > d_n \end{cases} \quad (7)$$

We therefore obtain a system of n equations to be checked for the feasibility conditions, with among other things the consideration of the occupancy time of the machine for each of the parts.

3.3 Stock dynamics

In a generalized way we can note the dynamics of the stock as follows:

$$\frac{dx_i(t)}{dt} = u_i(t) - d_i \qquad \forall i \in [1;n] \quad (8)$$

3.4 Instant cost

In the instantaneous cost, if we refer to the stated assumptions, we do not consider a cost related to modes, only the costs of inventorying, shortage and transition from the state of failure to the operational state.

$$g(\alpha, x_1, x_2, \dots, x_n, u_1, u_2, \dots, u_n, u_r) = \sum_{i=1}^{n} [c_i^+ x_i^+ + c_i^- x_i^-] + c_r u_r Ind\{\alpha = 0\} \quad (9)$$

With $x_i^+ = \max(0, x_i)$; $x_i^- = \max(-x_i, 0)$ et c_r, c^+, c^- the constants reflecting respectively the cost of repair, the cost of inventory and the cost of shortage. When implementing digital resolution with MATLAB it will be necessary to ensure that the cost of shortage is much higher than the cost of inventorying, otherwise the optimal policy will be to produce nothing.

3.5 Discounted total cost

$$J(\alpha, x_1, x_2, \dots, x_n, u_1, u_2, \dots, u_n, u_r) = E\left\{ \int_0^\infty e^{-\rho t} g(.) \,|\, x_i(0) = x_{i0}, \alpha(0) = \alpha_0 \right\} \quad \forall i \in [1;n] \quad (10)$$

3.6 Value function

$$v(\alpha, x_1, \dots, x_n) = \inf_{(u,u_r) \in \Gamma(\alpha)} J(\alpha, x_1, x_2, \dots, x_n, u_1, u_2, \dots, u_n, u_r) \quad (11)$$

4. Hamilton-Jacobi-Bellman equations

Since the general development of the equations of HJB is already known, we can write directly its generalized expression:

$$\rho v(\alpha, x_1, \dots, x_n) = \min_{(u,u_r) \in \Gamma(\alpha)} \left[\begin{array}{c} g(.) \\ + \sum_{\beta \in B} q_{\alpha\beta}(.) v(\beta, x_1, \dots, x_n) \\ + \sum_{i=1}^{n} \frac{\partial v(\alpha, x_1, \dots, x_n)}{\partial x_i} . \frac{dx_i}{dt} \end{array} \right] \quad (12)$$

4.1 Kushner's approach – numerical resolution

We will solve this problem using Kushner's approach, so we can approximate the value function and its derivative numerically. Note h_{x_i} the step on the stock x_i, by definition of discretization (finite differences) we have:

$$\frac{\partial v(\alpha, x_1, \dots, x_i, \dots, x_n)}{\partial x_i} = \begin{cases} \frac{1}{h_{x_i}} \begin{bmatrix} v^h(x_1, \dots, x_i + h_{x_i}, \dots, x_n) \\ -v^h(\alpha, x_1, \dots, x_n) \end{bmatrix} & si\ u_i - d_i \geq 0 \\ \frac{1}{h_{x_i}} \begin{bmatrix} v^h(\alpha, x_1, \dots, x_n) \\ -v^h(x_1, \dots, x_i - h_{x_i}, \dots, x_n) \end{bmatrix} & sinon \end{cases} \quad (13)$$

By replacing expression 13 in HJB equation:

$$\rho v^h(\alpha,.) = \min_{(u,u_r)\in\Gamma(\alpha)} \begin{bmatrix} g(.) + q_{\alpha\alpha} v^h(\alpha,.) + \sum_{\substack{\beta\in B \\ \beta\neq\alpha}} q_{\alpha\beta}(.) v^h(\beta,.) \\ + \sum_{i=1}^{n} \frac{u_i - d_i}{h_{x_i}} \begin{bmatrix} v^h \begin{pmatrix} x_1, \dots, x_i \\ +h_{x_i}, \dots, x_n \end{pmatrix} \\ Ind\{u_i - d_i \geq 0\} \\ +v^h \begin{pmatrix} x_1, \dots, x_i \\ -h_{x_i}, \dots, x_n \end{pmatrix} \\ Ind\{u_i - d_i < 0\} \end{bmatrix} \\ - \sum_{i=1}^{n} \frac{|u_i - d_i|}{h_{x_i}} v^h(\alpha,.) \end{bmatrix} \quad (14)$$

We factor on the left member and then by dividing:

$$v^h(\alpha,.) = \min_{(u,u_r)\in\Gamma(\alpha)} \begin{bmatrix} \frac{g(.) + \sum_{\substack{\beta\in B \\ \beta\neq\alpha}} q_{\alpha\beta}(.) v^h(\beta,.)}{\left(\rho + \sum_{i=1}^{n} \frac{|u_i - d_i|}{h_{x_i}} + |q_{\alpha\alpha}|\right)} \\ + \sum_{i=1}^{n} \frac{u_i - d_i}{h_{x_i}} \begin{bmatrix} v^h \begin{pmatrix} x_1, \dots, x_i \\ +h_{x_i}, \dots, x_n \end{pmatrix} \\ Ind\{u_i - d_i \geq 0\} \\ +v^h \begin{pmatrix} x_1, \dots, x_i \\ -h_{x_i}, \dots, x_n \end{pmatrix} \\ Ind\{u_i - d_i < 0\} \end{bmatrix} \end{bmatrix} \quad (15)$$

Ask :

$$\Omega_h^\alpha = \sum_{i=1}^{n} \frac{|u_i(t) - d_i|}{h_{x_i}} + |q_{\alpha\alpha}|$$

$$p_{x_i}^+(\alpha) = \begin{cases} \frac{u_i(t) - d_i}{h_{x_i} \Omega_h^\alpha} \\ si\ u_i(t) - d_i > 0 \\ 0 \quad otherwise \end{cases}$$

$$p^\beta(\alpha) = \frac{q_{\alpha\beta}(.)}{\Omega_h^\alpha}$$

$$p_{x_i}^-(\alpha) = \begin{cases} \frac{d_i - u_i(t)}{h_{x_i} \Omega_h^\alpha} \\ si\ u_i(t) - d_i \leq 0 \\ 0 \quad otherwise \end{cases} \quad (16)$$

In this way, we obtain:

$$v^h(\alpha,.) = \min_{(u,u_r)\in\Gamma(\alpha)} \begin{bmatrix} \frac{g(.)}{\Omega_h^\alpha \left(1 + \frac{\rho}{\Omega_h^\alpha}\right)} \\ + \frac{1}{1 + \frac{\rho}{\Omega_h^\alpha}} \begin{bmatrix} \sum_{\beta\neq\alpha} p^\beta(\alpha) v^h(\beta,.) \\ + \sum_{i=1}^{n} p_{x_i}^{\pm} v^h \begin{pmatrix} x_1, \dots, x_i \\ \pm h_{x_i}, \dots, x_n \end{pmatrix} \end{bmatrix} \end{bmatrix} \quad (17)$$

In the case study in particular, we will solve an M1P2 system, which would give us the following set of equations :

$$\begin{cases} v^h(0,.) = \min_{(u,u_r)\in\Gamma(\alpha)} \begin{bmatrix} \frac{c_1^+ x_1^+ + c_1^- x_1^- + c_2^+ x_2^+ + c_2^- x_2^- + c_r u_r}{\Omega_h^0 \left(1 + \frac{\rho}{\Omega_h^0}\right)} \\ + \frac{1}{1 + \frac{\rho}{\Omega_h^0}} \begin{bmatrix} p^1(0) v^h(1,.) \\ + p_{x_1}^- v^h(x_1 - h_{x_1}, x_2) \\ + p_{x_2}^- v^h(x_1, x_2 - h_{x_2}) \end{bmatrix} \end{bmatrix} \\ v^h(1,.) = \min_{(u,u_r)\in\Gamma(\alpha)} \begin{bmatrix} \frac{c_1^+ x_1^+ + c_1^- x_1^- + c_2^+ x_2^+ + c_2^- x_2^-}{\Omega_h^1 \left(1 + \frac{\rho}{\Omega_h^1}\right)} \\ + \frac{1}{1 + \frac{\rho}{\Omega_h^1}} \begin{bmatrix} p^0(1) v^h(0,.) \\ + p_{x_1}^{\pm} v^h(x_1 \pm h_{x_1}, x_2) \\ + p_{x_2}^{\pm} v^h(x_1, x_2 \pm h_{x_2}) \end{bmatrix} \end{bmatrix} \end{cases} \quad (18)$$

Resolution algorithm: To solve this optimization model, we will proceed by recurrence with a fixed precision criterion. We proceed in 4 steps:

1. Initialization
2. Iteration
3. Calculating the value function to get the order policy
4. Convergence test

Here are the detailed steps:

1. Initialization

We choose $\delta \in \mathbb{R}^{+*}$ the accuracy to be achieved in the convergence of the approximate value function.
With : $m = 1$ and we initialize the value function:

$$(v^h(\alpha, x_1, x_2)^m = 0 \quad \forall \alpha \in B \quad (19)$$

2 Iteration

$$(v^h(\alpha, x_1, x_2)^{m-1} = (v^h(\alpha, x_1, x_2)^m \quad (20)$$

3 Calculating the value function

$$T(v^h(\alpha, x_1, x_2)^{m-1} = (v^h(\alpha, x_1, x_2)^m = T^*(v^h(\alpha, x_1, x_2)^{m-1} \quad (21)$$

4 Convergence test

We calculate:

$$\overline{c} = \min_{(x_1,x_2)} \{(v^h(\alpha, x_1, x_2)^m - (v^h(\alpha, x_1, x_2)^{m-1}\} \quad (22)$$

$$\underline{c} = \max_{(x_1,x_2)} \{(v^h(\alpha, x_1, x_2)^m - (v^h(\alpha, x_1, x_2)^{m-1}\} \quad (23)$$

Then :

$$\begin{cases} c_{min} = \dfrac{\rho}{1-\rho}\overline{c} \\ c_{max} = \dfrac{\rho}{1-\rho}\underline{c} \end{cases} \quad (24)$$

Then if $|c_{min} - c_{max}| \leq \delta$ we stop the search because we consider to have found the numerical optimum. Otherwise we increment m and we loop.

The whole point of writing this algorithm lies in its application with the MATLAB environment, avoiding as much as possible adding complexity in the code that could lengthen the computation time. Indeed, the complexity of such a model can increase very quickly with the number of dimensions of production policy. In this case we have three decision variables so a problem with three (3) dimensions, and the calculation times are already of the order of 30s observable. We will talk about this again during the discussion on possible extensions.

5. Numerical Example and Results Analysis

We present in Table II, the results obtained with the MATLAB code for the choice of following starting parameters :

Table 2: Data of the problem (economic and technical parameters)

h_{x_1}	h_{x_2}	$\overline{u_1}$	$\overline{u_2}$	$MTTR_{max}$	$MTTR_{min}$
0.5	0.5	0.25	0.3	50	10
q_{10}	$\underline{u_r}$	$\overline{u_r}$	d_1	d_2	ρ
0.1	0.02	0.1	0.25	0.4	0.1
c_1^+	c_1^-	c_2^+	c_2^-	c_r	τ_1
1	30	4	30	100	0.1
τ_2	$x_{1,inf}$	$x_{1,sup}$	$x_{2,inf}$	$x_{2,sup}$	δ
0.1	-5	20	-5	20	0.001

We expected to find policies of pace « Hedging Point Policy » in view of the way of considering the values that can take u_1, u_2 et u_r in this case study. This is indeed what we observe in the results that follow. Before displaying these results, remember that the form of the value functions for this specific case is as follows:

$$\begin{cases} v^h(0,.) \\ = \min\limits_{(u,u_r)\in\Gamma(\alpha)} \left[\dfrac{c_1^+x_1^+ + c_1^-x_1^- + c_2^+x_2^+ + c_2^-x_2^- + c_r u_r}{\Omega_h^0\left(1+\frac{\rho}{\Omega_h^0}\right)} + \dfrac{1}{1+\frac{\rho}{\Omega_h^0}} \begin{bmatrix} p^1(0)v^h(1,.) \\ +p_{x_1}^- v^h(x_1 - h_{x_1}, x_2) \\ +p_{x_2}^- v^h(x_1, x_2 - h_{x_2}) \end{bmatrix} \right] \\ v^h(1,.) \\ = \min\limits_{(u,u_r)\in\Gamma(\alpha)} \left[\dfrac{c_1^+x_1^+ + c_1^-x_1^- + c_2^+x_2^+ + c_2^-x_2^-}{\Omega_h^1\left(1+\frac{\rho}{\Omega_h^1}\right)} + \dfrac{1}{1+\frac{\rho}{\Omega_h^1}} \begin{bmatrix} p^0(1)v^h(0,.) \\ +p_{x_1}^{\pm} v^h(x_1 \pm h_{x_1}, x_2) \\ +p_{x_2}^{\pm} v^h(x_1, x_2 \pm h_{x_2}) \end{bmatrix} \right] \end{cases} \quad (25)$$

All the parameters previously chosen respect the tacit condition of superiority of the cost of scarcity vis-à-vis the cost of inventorying, otherwise it is obvious that the production solution would simply be to produce nothing and therefore not to store anything. Note that we checked the feasibility of the mathematical model upstream of the resolution/optimization loop to avoid optimizing a system that would not be physically feasible. Indeed, it would be a waste of time. We chose to give different storage costs for the two parts, as well as different demands and max production ratio.

Indeed the type 2 part is more expensive to store but it is more in demand than the type 1 part in the customer market. The FMS is forced to produce it but the optimization will be all the more interesting in this case to avoid too high costs.

The results obtained show that the optimal policy is a critical threshold policy:

$$u_1(\alpha = 1, x_1, x_2) = \begin{cases} \overline{u_1}\ si\ x_1 < Z_1 \\ d_1\ si\ x_1 = Z_1 \\ 0\ \ si\ x_1 > Z_1 \end{cases} \quad (26)$$

$$u_2(\alpha = 1, x_1, x_2) = \begin{cases} \overline{u_2}\ si\ x_2 < Z_2 \\ d_2\ si\ x_2 = Z_2 \\ 0\ \ si\ x_2 > Z_2 \end{cases} \quad (27)$$

Knowing that Z_1, Z_2 designate the critical thresholds, in mode 1 (only mode of production) on stocks 1 and 2 respectively, of the overall production policy (two pieces). As we can see in Figure 3, $Z_1 = 5$, which means that in the area where the stock 1 is less than 5 the machine must produce the type 1 part at the maximum production rate. At the limits of this zone the type 1 part is produced at the rate of demand, and outside this zone it is not produced at all. The value of the Hedging Point varies a little with stock 2, on average it is equal to about 5.5.

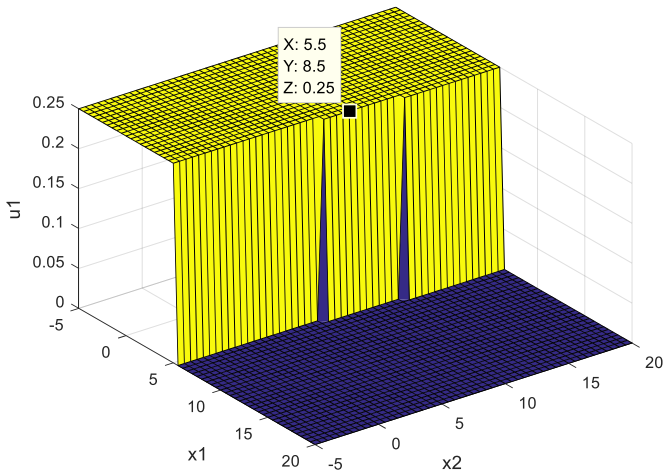

Figure 3. Production policy part 1

Similarly, for the production policy of Exhibit 2, Figure 4, we obtain that $Z_2 = 4$, which means that if the stock 2 is less than 4 we must produce the type 2 part at the maximum production rate, at the demand rate if in limit, then no longer produce it if we have more than 4 in stock.

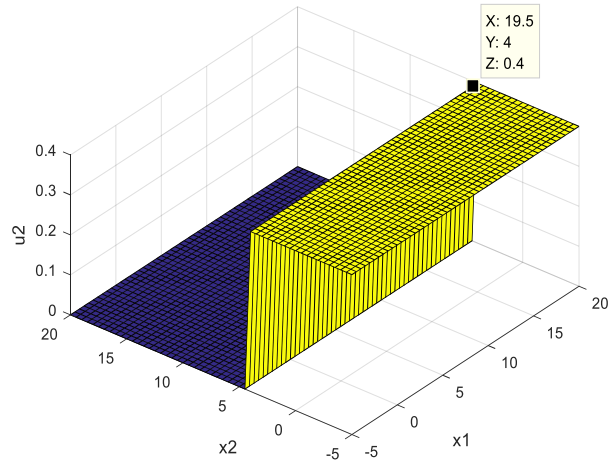

Figure 4. Production policy part 2

Then comes the corrective maintenance check. We find that the policy, Figure 5, is to replace it at the maximum speed (maximum rate) when the stock of Part 1 is less than 2 ($=Z_{31}$) or if the stock of part 2 is less than 3.5 ($=Z_{32}$). Otherwise to minimize the replacement cost in the overall context, repairs are made at the minimum speed (minimum ratio).

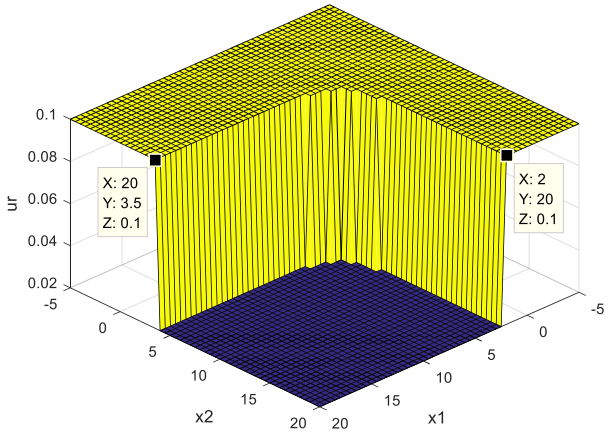

Figure 5. Repair policy

As for the appearance of the value functions, Figures 6 and 7 associated, here is quickly their appearance, we will not use them for sensitivity analysis, we simply present them here to check the form of the policies previously obtained. And indeed to minimize the cost it is better to move away from areas of scarcity.

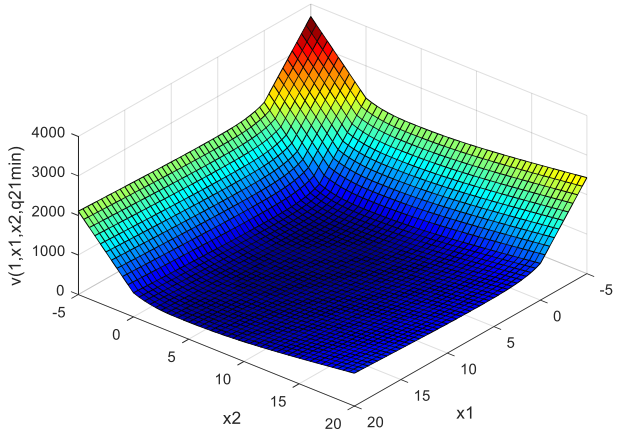

Figure 6. Value function at mode 1

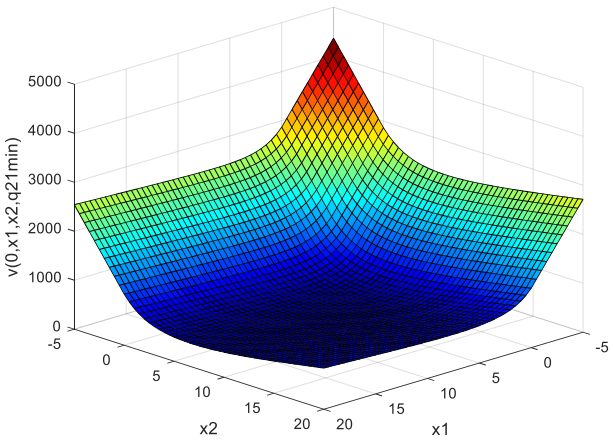

Figure 7. Value function at mode 0

Sensitivity analysis : During a sensitivity analysis, the costs defined in the instantaneous cost function are mainly used by observing the effect on the values of the critical thresholds. We could also conduct a sensitivity analysis on parameters that are more related to the definition of the production unit itself (such as the failure rate, the maximum production ratio, etc.), but in this case we will only compare according to the fluctuation of costs, since we have assumed by empirical observation to know the behavior of the machine, which should therefore not change in this case.

To do this, we will vary each cost constant independently and observe the impact of this variation on the critical production and maintenance thresholds. The plan of the sensitivity analysis is available in its full version, here we will detail, on a case-by-case basis, what the variations are and explain them in relation to the real phenomena that occur.

Case n°1 : Change in the cost of inventorying the product 1

When the cost of inventorying (MEI) of product 1 c_1^+ increases, the optimal production threshold decreases, which is logical since the higher the storage cost, the less storage is needed to decrease total costs. We then agree to get closer to the shortage to avoid too expensive storage. This is what we can observe on the following curve (Figure 8), built from the program we have written:

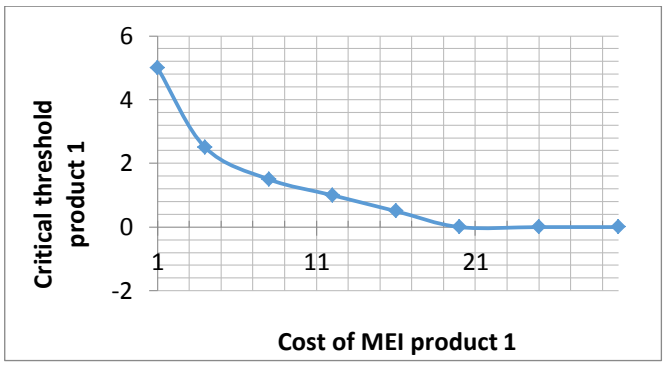

Figure 8. Change in critical threshold Z1 as a function of the MEI cost of the type 1 product

Note that the value tends to balance around the zero stock, because the cost of shortage is also important so we can not

afford to get there, the limit becomes critical since a failure that would occur with such a policy could collapse the system depending on the time that the repair will take.

Case n°2 : Change in product inventory cost 2

Overall we obtain the same behavior, with different threshold values however since the demand rates of the two products are different. We can observe this on the curve in Figure 9 below :

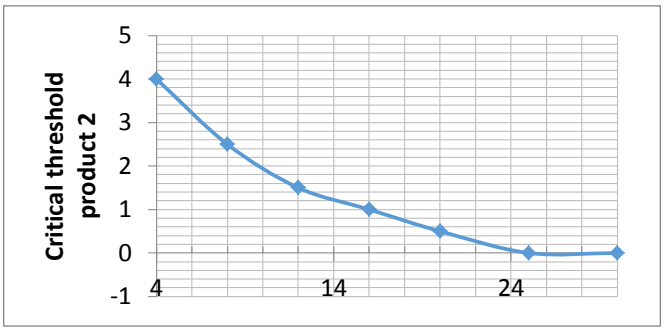

Figure 9: Change in Z2 Critical Threshold based on the MEI cost of the Type 2 product

Case n°3 : Change in product shortage cost 1

In Figure 10, increasing the cost of scarcity is tantamount to forcing the average stock over time to increase. In other words, we are trying to make the stock always more full. This tends to show that the higher this cost, the higher the critical threshold will be, until the user-defined stock frontier value is reached. Indeed, the more the cost of shortage increases, the more the minimum value of the value function will move to large stocks., with x_{sup} for asymptote limit.

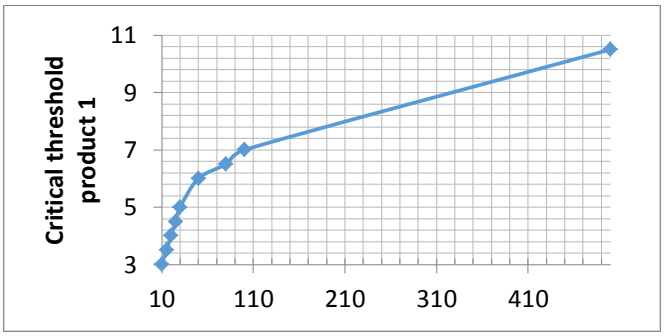

Figure 10: Change in critical threshold Z1 as a function of the cost of shortage of the type 1 product

In addition, we also observe another phenomenon associated with the variation in the cost of product shortage 1, the variation of the maintenance policy on the stock x_1. Indeed the critical threshold Z_{31} also increases with the cost of shortage. This is normal since the more this cost increases, the more we need to produce at a large stock, which means that we must quickly repair the machine if it breaks down while the stock is in the area where we should produce, otherwise we will see the stock fall and then find ourselves in shortage, where the cost is very high. It also tends towards the maximum stock of product 1, which is logical in view of our previous definitions.

Case n°4: Change in product shortage cost 2

We obtain results similar to the previous case except that it is now at the stock level of product 2 that the change occurs.

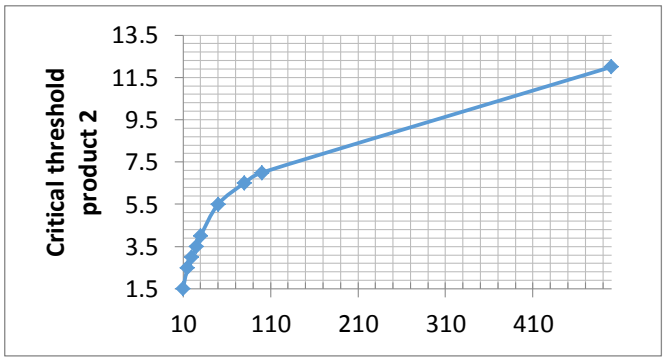

Figure 11: Change in critical threshold Z2 as a function of the cost of shortage of the type 2 product

In Figure 11we also observe the phenomenon of displacement of the critical threshold of maintenance planning Z_{32} on stock 2 for the same reasons as listed in the previous case.

Case n°5: Change in repair cost

The latter case of sensitivity analysis consists in varying the cost of repairing the machine. This cost, as we have seen in the development part of the equations of HJB, only intervenes in the equation of the failure mode 0. We expect that the variation will not change production policies since it is not the mode of production that is impacted. Indeed during the tests, it turns out that Z_1 and Z_2 do not vary with the increase in c_r, rather, they are Z_{31} and Z_{32} which will vary. We can observe with the graphs in Figures 12 and 13 below that when the cost of repair increases, the critical maintenance thresholds both decrease to an asymptotic value. This means that the higher the repair cost, the more critical it is to perform this repair in the right stock.

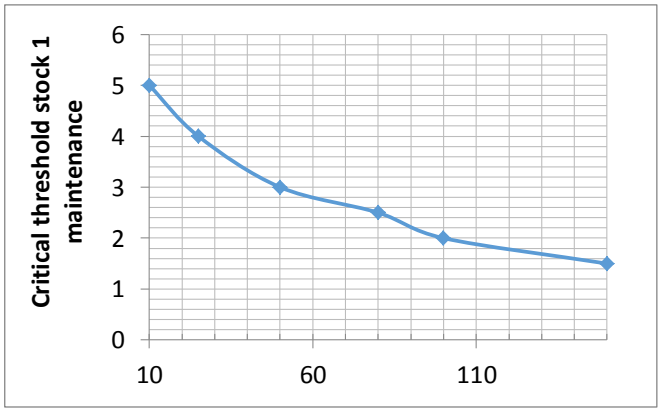

Figure 12: Change in critical threshold Z31 as a function of repair cost

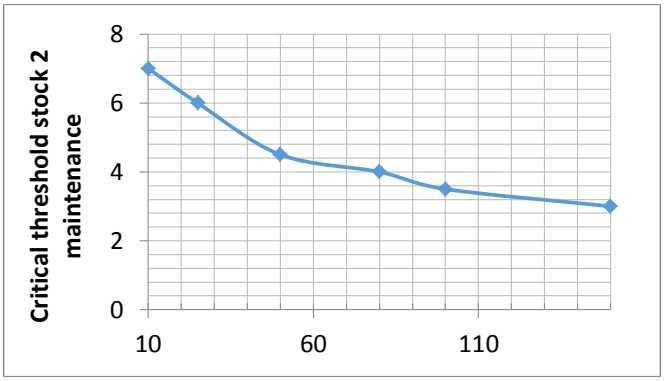

Figure 13 Change in critical threshold Z31 as a function of repair cost

In more detail, when the cost of corrective maintenance is not high, we can repair at the maximum speed (= maximum ratio) in more cases since it does not cost much. Indeed, in view of the definition we have given to the cost of repair, when the restoration is done at the maximum rate it costs more than the minimum rate since the cost corresponds to the product of c_r and transition rates. So when c_r increases, repair at the maximum rate becomes more expensive, and one can no longer afford to repair at the maximum speed if the stock is sufficient to take this failure. In this case we prefer to repair at the minimum rate, which is why the critical threshold decreases. The limit will be reached when the repair is necessary to avoid a shortage cost too high, we will then be forced to do the repair at the max rate, even if it is expensive, since the shortage would cost even more. However, it depends on the situation, if in one case the cost of shortage remains lower than the cost of repair, then the asymptotic value will be even lower, tending towards the minimum stock.

6. Conclusion

This paper has allowed us to introduce corrective maintenance strategies based on the number of setups. We considered a single-machine system capable of producing only one type of product at a time of finished products with a significant setup time and cost. Such a machine is prone to random failures and repairs. We have found a new control law structure that allows joint control of production, setup, and corrective maintenance. This work aimed to study the effect of a wide range of system configurations on the optimal parameters of the various control policies considered, minimizing the total cost incurred. The complexity of the problem has led us to analyze simpler situations up to the consideration of the interactions of the control policies presented. The policies obtained are of the type of modified hedging point policy, given that they depend on the number of setup activities. The results in this paper make a significant contribution to the control literature of production systems due to the fact that such a problem has never been addressed. Another possible extension concerns random requests, to implement such a model we could operate as we did with the corrective maintenance rate, namely that the random request rate varies between two min and max values. These values would be taken from observing the mean and standard deviation of variable demand from any manufacturer. Nevertheless, to consider a random request rate it seems more complex than our case. Since the dynamics of the stock depend directly on the rate of demand, this means that a way must be found to relate demand to production.

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